Statistical Analysis of a Communication System Based on the Periodic Nonlinear Fourier Transform

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Abstract: We analyse the statistical properties of the nonlinear spectrum in optical fiber influenced by ASE noise and show that the Gaussian distribution assumption for the effective noise in nonlinear Fourier domain is valid.
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1. Introduction

One of the new methods recently employed for the nonlinearity mitigation in fiber-optic communication systems is the inverse scattering transform (IST), also known as the nonlinear Fourier transform (NFT) [1, 2]. NFT (or IST) is the method providing analytical solutions of integrable evolutionary equations. In particular, it is applicable to the the nonlinear Schrödinger equation (NLSE), $iq_z - \frac{\beta_2}{2}q_{tt} + \gamma q|q|^2 = \nu$ as the master model governing the light envelope, $q(t,z)$, evolution in the fiber where ν is the ASE noise. Similar to the Fourier transform, the NFT transfers the NLSE into another space, decomposing the time-domain waveform into the so-called nonlinear spectrum (NS). In this paradigm, instead of solving the NLSE, one has to find the solution of an associated differential equation, Zakharov-Shabat system (ZSS) in which the solution to the NLSE acts as a potential and the eigenvalues of the system, λ, remain invariant along the evolution direction, z [2]. IST decomposes the solution into some nonlinear modes, the most well known form of which are famous solitons. Not only the NS can visualize some hidden characteristics of the solution of the NLSE, but it can also be used to convey data from transmitter to receiver due to the invariance of some of its components (eigenvalues). Since all communication systems suffer from random perturbations either coming from amplifiers, ASE, or emerging due to the imperfections of the receiver, it is of great interest to analyse the stochastic characteristics of the NS under the action of noise influence. The discrete components of NS represent the solitonic solutions of the NLSE, and their statistical behavior is well studied: A famous example is the Gordon-Haus effects describing the soliton-noise beating resulting in the random walks (jitters) of the soliton parameters [3].
To apply IST in solving NLSE, depending on the system symmetry, one can assume the solution to be periodic in time or decay as time goes to infinity [2]. The assumption of the periodic solutions (periodic signal continuation) brings about some extra advantages for the communication system design [4]. For the periodic IST (PNFT), the NS consists of two parts: The *main spectrum* which is the invariant part, and the *auxiliary spectrum* which is the dynamic part. Here for the sake of simplicity, we only take into account the main spectrum which comprises finitely many complex points. This concept can be applied in communication system in the following way: at the transmitter, one picks an arbitrary complex number and constructs a solution to NLSE which acquires this complex value as its main spectrum. This signal is being sent to the fiber which at the receiver the main spectrum of the received signal (supposedly the same as the transmitted one) is calculated via the so-called *direct transform*. The obtained main spectrum at the receiver is used to retrieve the initial encoded data. Due to ASE noise and inverse and direct transformation inaccuracies, the received main spectrum is expected to be distorted. In this work we study the structure of the NS distortion which should help in the design of detection and coding schemes for NFT-based communication systems.

2. Signal construction

There is still a lack of an efficient numerical method to construct signal from an arbitrary main spectrum. Thus, to study the stochastic behavior of the main spectrum we chose a more simple two-phase signal in which we can easily set a specific eigenvalue in the main spectrum and construct the resulting signal [5]. The signal is given by the expression

$$q(t,z) = A\frac{\cosh(\phi z - i\sigma) + B\cos(\xi t - \alpha)}{\cosh\phi z + B\cos(\xi t - \alpha)}e^{iNz}, \tag{1}$$

where all parameters are calculated according to the chosen purely imaginary eigenvalue λ_1 (see [4,5] for more details).

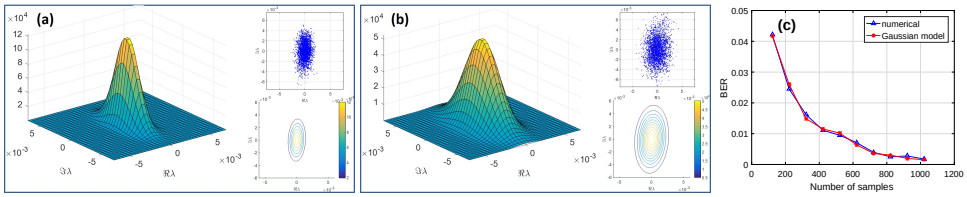

Fig. 1: a) PDF for $\lambda = 0.9i$, b) PDF for $\lambda = 1.1i$, and c) BER against number of samples for Gaussian assumption and numerically obtained from simulation at $z = 1000$ Km, constellation size of $K = 32$ and signal power of $P = 0.5$ dBm.

3. Simulation results

We send our signal into a 1000 Km fiber with $\beta_2 = -21.67\,\mathrm{ps}^2/\mathrm{km}$, $\gamma = 1.27\,\mathrm{W}^{-1}\mathrm{km}^{-1}$, assuming ideal distributed Raman amplification (resulting in the distributed noise) to overcome loss of $\alpha \approx 0.2$ dB/km. We choose the main spectrum points values $\lambda_1 = 0.9i$ and $\lambda_2 = 1.1i$, and calculate the histogram of the imaginary and real part of the received points. The empirical conditional PDFs of displacement of the received eigenvalues on the transmitted one is shown in Fig. 1 for λ_1 and λ_2; the number of samples in these plots was $N = 522$. In the insets of these figures we also depicted the received main spectrum. It is clear from these figures that the received eigenvalues are distributed in a larger area for the point having a bigger imaginary part. This occurred because of the dependency of the numerical method that we used to calculate the main spectrum (the Ablowitz-Ladik method, see [2]) on the imaginary part of the point. Since we do not have any analytical information about the distribution of λ, a general nonparametric representation is fitted to our data with a smoothing kernel of the normal type and a chosen bandwidth in a way that it can capture the changes in the histogram. We compare different known PDFs with this nonparametric one and find the one with the least difference. The PDF which fits the best to the kernel based estimation is a Normal distribution. In a communication system in which data is drawn from a one dimensional constellation along the imaginary axis in a confined area, the most problematic points are the largest ones with the largest span. Thus we can calculate a lower bound for the error in such a system by assuming a Normal distribution with estimated parameters for two largest points in a constellation with size $K = 32$. In Fig. 1 (c) we depict the bit error rate (BER) calculated by Gaussian assumption analytically and the BER obtained by counting the mismatches between the transmitted and received symbol streams against the number of samples. As is evident from Fig. 1 (c), the Normal distribution assumption is valid since two curves match well. This gives rise to the idea of using pilots to equalize the system in which, thanks to the time invariance of the optical fiber link, one can estimate the deterministic distortions and compensate it at the receiver.

4. Conclusion

After constructing a signal with the desired main spectrum, we get the empirical PDF of the received eigenvalues by sending many symbols into a fiber link of length 1000 Km. Based on the PDFs obtained, we calculate the BER and compare it with the BER coming from directly counting the mismatches between transmitted and received symbols. These simulation results show the eligibility of assuming Gaussian distribution for noise in nonlinear Fourier domain.

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