Comparison of Numerical Bit Error Rate Estimation Methods in 112Gbs QPSK CO-OFDM Transmission

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Abstract *We demonstrate an accurate BER estimation method for QPSK CO-OFDM transmission based on the probability density function of the received QPSK symbols. Using a 112Gbs QPSK CO-OFDM transmission as an example, we show that this method offers the most accurate estimate of the system's performance in comparison with other known approaches.*

Introduction

Coherent optical orthogonal frequency division multiplexing (CO-OFDM) is an attractive transmission technique providing efficient way to mitigate for both chromatic and polarization-mode dispersion in fiber links[1]. Design, development, and operation of CO-OFDM systems require simple, efficient and reliable methods of their performance evaluation. The bit error rate (BER) in CO-OFDM systems can be estimated using Monte Carlo simulation with direct error counting. However, this method relies on a large number of statistical samples and, in general, is time-consuming. Therefore, it is practically important to develop efficient indirect numerical and statistical methods for evaluating CO-OFDM system performance. For coherent communication systems, the error vector magnitude (EVM) is commonly used as a fast measure of the received digital signal's quality. Few other relevant methods of evaluating the signal quality have recently been proposed and experimentally verified and compared[2-4]. A modification of standard EVM was introduced to estimate the performance of CO-OFDM systems[5]. However, the exact relationship between the BER and the EVM in CO-OFDM still remains an open problem. In addition, to the best of our knowledge, the relative performance of different BER estimation methods for coherent QPSK systems has never been compared for CO-OFDM.

The purpose of this paper is twofold. Firstly, we introduce a novel BER estimation method for CO-OFDM systems. Then we compare the performance of the proposed method with other common BER estimators.

To have a correct reference points, we compute BER through direct error-counting. The directly measured BER, then, is converted to an equivalent "Gaussian noise" Q-factor in dB using the expression (here - definition of Q):

$$Q_{BER} = 20\log[\sqrt{2}\cdot efrc^{-1}(2BER)]$$

where $erfc^{-1}$ is the inverse complementary error function. This sets the reference Q-factor used in evaluation of different indirect methods.

Error vector magnitude (EVM): The EVM is defined as the root-mean-square value of the difference between a collection of the measured and the ideal symbols. The difference is normalized by the average power per symbol in the constellation.

$$EVM = \sqrt{\frac{1/N\left(\sum_{k=1}^{N}\left|c_k - c_{k,ideal}\right|^2\right)}{1/N\left(\sum_{k=1}^{N}\left|c_{k,ideal}\right|^2\right)}}$$

c_k is the k'th received symbol and $c_{k,ideal}$ is the corresponding ideal constellation point. The BER can be evaluated from the EVM by:

$$BER = \frac{1}{2}erfc(\frac{EVM^{-1}}{\sqrt{2}})$$

Q-factor 1 (Q1): The Q-factor in QPSK CO-OFDM systems can also be estimated by the following formula used in Ref. 5:

$$Q_1 = \left(\sqrt{\frac{\sum\left|c_{k,i} - \langle c_k\rangle_{(i)}\right|^2}{\sum\left|\langle c_k\rangle_{(i)}\right|^2}}\right)^{-1}$$

where $\langle c_k\rangle_{(i)}, i = 1,2,3,4$ are the means of the received symbols $c_{k,i}$ that fall into the i'th quadrant of the constellation diagram.

Q-factor 2 (Q2): The Q-factor can be evaluated from in-phase and quadrature components of the received QPSK signal as follows[2]:

$$Q_{\mathrm{Re}} = \frac{\langle c_{k,\mathrm{Re}}(c_{k,\mathrm{Re}} > 0)\rangle - \langle c_{k,\mathrm{Re}}(c_{k,\mathrm{Re}} < 0)\rangle}{\sigma_{\mathrm{Re}}(c_{k,\mathrm{Re}} > 0) + \sigma_{\mathrm{Re}}(c_{k,\mathrm{Re}} < 0)}$$

$$Q_{\mathrm{Im}} = \frac{\langle c_{k,\mathrm{Im}}(c_{k,\mathrm{Im}} > 0)\rangle - \langle c_{k,\mathrm{Im}}(c_{k,\mathrm{Im}} < 0)\rangle}{\sigma_{\mathrm{Im}}(c_{k,\mathrm{Im}} > 0) + \sigma_{\mathrm{Im}}(c_{k,\mathrm{Im}} < 0)}$$

$\langle\cdot\rangle$ and $\sigma(\cdot)$ denote the mean and the standard deviation (STD).

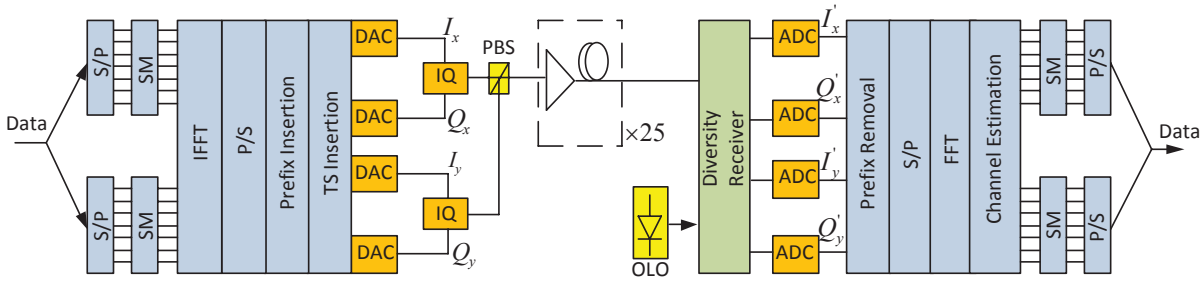

Fig. 1: Block diagram of PDM CO-OFDM system. S/P: serial/parallel conversion, P/S: parallel/serial conversion, SM: symbol mapping, TS: training symbols, DAC: digital-to-analog converter, I/Q: I/Q modulator, PBS: polarization beam spliter, OLO: optical local oscilator

The BER is obtained by averaging the estimations from both Q_{Re} and Q_{Im}

$$BER = \left\langle \frac{1}{2} erfc\left(\frac{Q_{\mathrm{Re}}}{\sqrt{2}}\right), \frac{1}{2} erfc\left(\frac{Q_{\mathrm{Re}}}{\sqrt{2}}\right) \right\rangle$$

Q-factor 3 (Q3): Another definition of Q-factor was introduced in Ref. 2 as the ratio between the mean and the STD value of each constellation point[2]. For the symbol in the first quadrant, the Q-factors are:

$$Q_{\mathrm{Re}} = \frac{\left|\left\langle c_{k,\mathrm{Re}}(c_{k,\mathrm{Re}} > 0, c_{k,\mathrm{Im}} > 0)\right\rangle\right|}{\sigma_{\mathrm{Re}}(c_{k,\mathrm{Re}} > 0, c_{k,\mathrm{Im}} > 0)}$$

$$Q_{\mathrm{Im}} = \frac{\left|\left\langle c_{k,\mathrm{Im}}(c_{k,\mathrm{Re}} > 0, c_{k,\mathrm{Im}} > 0)\right\rangle\right|}{\sigma_{\mathrm{Im}}(c_{k,\mathrm{Re}} > 0, c_{k,\mathrm{Im}} > 0)}$$

The overall BER can be averaged by both Q_{Re} and Q_{Im} for all the constellation symbols.

Simulation setup: The block diagram of a 112Gbs PDM CO-OFDM system is shown in Fig. 1. 112Gbs stream data is first divided into x- and y-polarization branches, each of which is then mapped onto 2048 subcarriers using QPSK modulation format with Gray code subsequently transferred to the time domain by an IFFT of size 4096 while zeros occupy the remainder. A cyclic prefix of length 512 is used to accommodate dispersion. The long-haul fiber link is assumed to consist of 25×80-km spans of standard single mode fiber (SSMF) with the loss parameter of 0.2dB/km and PMD coefficient of $1ps/\sqrt{Km}$. The fiber span loss is compensated by Erbium-doped optical amplifiers (EDFA) with 16dB of gain and a noise figure of 6dB. In simulation amplified spontaneous emission (ASE) noise is added inline. The transmitter laser and local oscillator laser were considered noiseless. The fiber nonlinearity coefficient and dispersion are $1.22W^{-1}km^{-1}$ and $16ps/nm/km$ respectively. The simulated time window contains 2^{18} samples and the total number of simulated data bits is approximately 2^{19}.

Received phase distribution in CO-OFDM

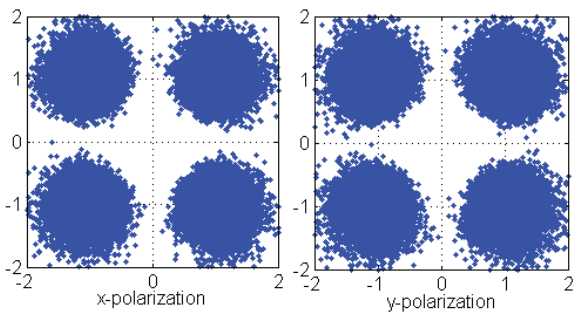

Fig. 2: Received QPSK constellation diagrams in x- and y-polarizations

In PDM CO-OFDM linear phase distortion due to fiber dispersion can be effectively compensated at the receiver through channel estimation. The channel response matrix of each subcarrier is calculated using training symbols[5]. Nonlinear phase noise caused by the interaction of ASE noise and fiber nonlinearity is dominated by the interaction of ASE noise and four-wave mixing (FWM)[6]. Therefore, the received phase in CO-OFDM has a Gaussian-like distribution due to the fact that independent data is carried on a large number of subcarriers, so the central limit theorem can be applied[7]. We verify this by studying the phase distribution in each quadrant of the received constellation of x- and y-polarizations.

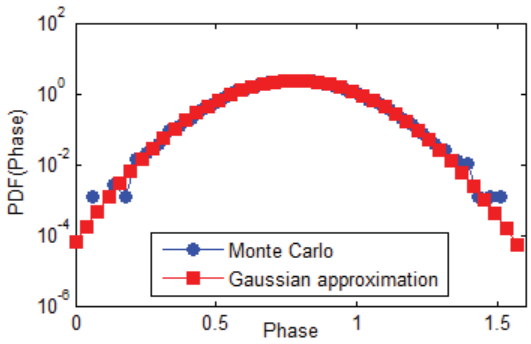

Fig. 3: Receivd phase distribution in the first quadrant, x-polarizarion, 0dBm of input power

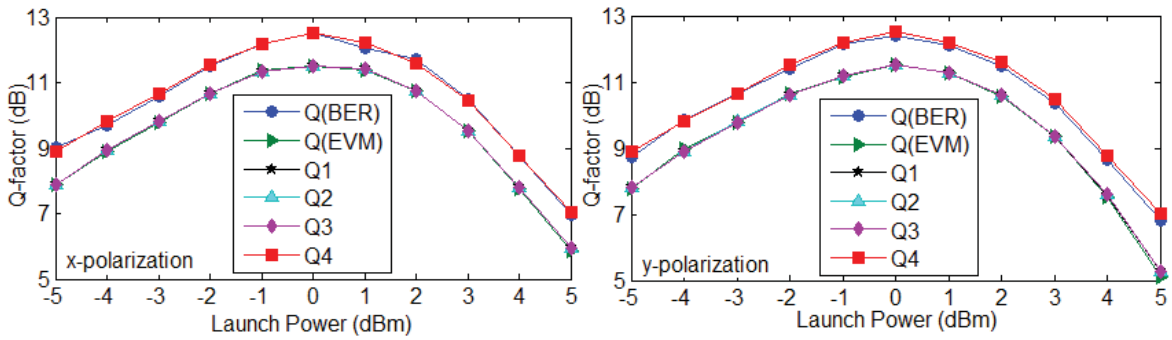

Fig. 4: Q-BER values as a function of the launch power for x- and y-polarization of 112Gbs PDM CO-OFDM after 2000km of transmission. Q(EVM), Q1, Q2, Q3 all follow the lower curves.

Figure 2 shows the received QPSK constellation diagrams for x- and y-polarizations after compensation of linear phase distortion caused by fiber dispersion and average nonlinear phase noise. The input power is 3dBm. The received QPSK symbols in each quadrant are then separated to investigate their phase distribution. Fig. 3 shows (in a logarithmic scale) a good agreement between the Gaussian approximation and the actual received phase distribution in the first quadrant, x-polarization. Similar results were obtained for all other quadrants of x-polarization and all four quadrants of y-polarization.

Using a Gaussian approximation, the PDF of the received phases (ϕ_k) in four constellation quadrants can be expressed as:

$$f_k(\phi) = \frac{1}{\sigma_k\sqrt{2\pi}} \cdot \exp(-\frac{(\phi-\overline{\phi_k})^2}{2\sigma_k^2})$$

where $f_k(\phi)$, $\overline{\phi_k}$, σ_k denote the PDF, means and standard derivations of the received phases in k'th quadrant ($k = 1...4$).In QPSK (Gray coded) CO-OFDM systems, information symbols can have one of the four following values:

$X_1 = \sqrt{2}\exp(j\pi/4)$, $X_2 = \sqrt{2}\exp(j3\pi/4)$,

$X_3 = \sqrt{2}\exp(-j3\pi/4)$, $X_4 = \sqrt{2}\exp(-j\pi/4)$

The error probability when X_1 is transmitted can be calculated as follows:

$$P_E(X_1) = \int_{-\infty}^{0} f_1(\phi)d\phi + \int_{\pi/2}^{+\infty} f_1(\phi)d\phi$$

$$= \frac{1}{2}\left[erfc(\frac{\overline{\phi_1}}{\sigma_1\sqrt{2}}) + erfc(\frac{\pi/2-\overline{\phi_1}}{\sigma_1\sqrt{2}}) \right]$$

where $erfc(x)$ stands for the complementary error function. Similarly, we can obtain expressions for $P_E(X_2), P_E(X_3), P_E(X_4)$, then the system's BER is given by:

$$BER = \frac{1}{8}\sum_{k=1}^{4}\left[erfc(\frac{\overline{\phi_k}-\arg(X_k)+\pi/4}{\sigma_k\sqrt{2}}) + erfc(\frac{\arg(X_k)+\pi/4-\overline{\phi_k}}{\sigma_k\sqrt{2}}) \right]$$

This expression offers a relatively simple way to estimate the performance of a CO-OFDM system by calculating the means and STDs of the received phases in each quadrant of a constellation diagram. Figure 4 shows the comparision of different Q estimation methods of PDM CO-OFDM for x- and y-polarization. The blue line with circle markers (Q(BER)) is the reference result of the direct error counting from Monte Carlo simulations. The red line with square markers (Q4) shows the result obtained using the estimation method proposed here based on a Gaussian approximation of the phase noise statistics. It can be seen that this method offers the most reliable estimation of the system's performance. For both x- and y-polarization the mismatch between the Q-factors obtained by the Gaussian approximation method and direct error counting is below 0.2dB. In contrast, the other approaches including EVM, Q1, Q2, Q3 demonstrate similar performances for PDM CO-OFDM and all these methods tend to underestimate the actual system Q(BER) value. The mismatch in Q-factor can be as high as 1dB depending on the launch power.

Conclusion

We have introduced a novel BER estimation method for CO-OFDM systems using the QPSK modulation format. Through numerical modeling of 112Gbs PDM CO-OFDM we demonstrate that this method is more accurate compared to commonly used popular BER estimators.

Acknowledgements

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