Dispersion-dominated nonlinear fiber-optic channel

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We propose to apply a large predispersion (having the same sign as the transmission fiber) to an optical signal before the uncompensated fiber transmission in coherent communication systems. This technique is aimed at simplification of the following digital signal processing of nonlinear impairments. We derive a model describing pulse propagation in the dispersion-dominated nonlinear fiber channel. In the limit of very strong initial predispersion, the nonlinear propagation equations for each Fourier mode become local and decoupled. This paves the way for new techniques to manage fiber nonlinearity.

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Nonlinear propagation effects in optical fiber communications combined with dispersive signal broadening and additive noise, in general, make impossible simple analytical description of the system transfer function that would link the output (received) optical field to the input (transmitted) field. In mathematical terms, the fiber communication channel is described by the stochastic nonlinear partial differential equation(s) [1]. Therefore, analysis of statistical properties of the channel requires extensive numerical modeling. Lack of a simple channel model impedes both theoretical analysis of channel capacity and application of engineering techniques, signal processing, and coding algorithms available, for instance, for a seminal additive white Gaussian noise channel. Recent progress in high-speed optical fiber communications is based on the technology of coherent detection with digital signal processing (DSP) to mitigate transmission impairments (see, e.g., [2–5] and references therein). Knowledge of an optical phase (in addition to field intensity) after coherent detection dramatically changes the whole concept of compensation and management of impairments in fiber-optic communication. In particular, linear effects such as dispersion and polarization-mode dispersion can be compensated entirely at the coherent receiver. Electronic signal processing using backpropagation has also been applied to the compensation of nonlinear fiber impairments. However, the drawback of the methods of reconstruction of the transmitted data from the received signal based on nonlinear backward propagation is that they rely on computationally intensive techniques. Real-time nonlinear digital backward propagation is still a challenging problem. It would be vitally important for development of efficient DSP techniques capable to mitigate nonlinear impairments to have models (at least in some limits) of the nonlinear fiber channels, which can be treated analytically or semianalytically.

We propose here a transmission technique based on a very large dispersive stretching of the optical signal before uncompensated fiber transmission in coherent communication systems with the aim to simplify the following DSP. It is assumed here that, in principle, very large predispersion can be imposed in a deterministic way, without necessarily adding substantial loss (and consequently, leading to an additional optical noise). This can be done both by optical devices or through electronic signal preprocessing. We would like to stress that this is not a standard, well-known precompensation technique. Just the opposite, we propose to apply a large predispersion with the *same sign as the transmission link dispersion*. The key idea behind such initial predispersion is that the imposed fast spectral oscillations of the signal simplify its subsequent dynamics and make the propagation equations integrable as the highest spectral modes are effectively averaged out. The proposed scheme leads to a relatively simple channel description without the assumption that the level of nonlinearity is necessarily small; i.e., it works even beyond the applicability region of a quasi-linear regime. In the limit of a very large initial predispersion, we derive an exact analytical description of a transfer function for the optical field in a nonlinear fiber channel with uncompensated (in optical domain) dispersion. In this Letter we outline the basic features of the scheme and supply some initial numerical simulation to verify its validity. Further developments, including statistical properties and specific applications of the model, will be presented elsewhere.

The averaged evolution of the two orthogonal polarizations of the field envelope $[U_1(z,t), U_2(z,t)]$ of the optical field along the fiber is well approximated by the Manakov equations [1,5,6]:

$$\frac{\partial U_1}{\partial z} = -\frac{\alpha}{2}U_1 - i\frac{\beta_2}{2}\frac{\partial^2 U_1}{\partial t^2} + i\frac{8\gamma}{9}J \times U_1 + \eta_1,$$
$$\frac{\partial U_2}{\partial z} = -\frac{\alpha}{2}U_2 - i\frac{\beta_2}{2}\frac{\partial^2 U_2}{\partial t^2} + i\frac{8\gamma}{9}J \times U_2 + \eta_2, \qquad (1)$$

where $J = |U_2|^2 + |U_1|^2$ is the nonlinearity averaged over polarization inhomogeneities and β_2, γ, $\eta_{1,2}$, and α are the group velocity dispersion, nonlinear coefficient, distributed noise and linear loss, respectively [1]. We focus here on deterministic nonlinear dynamics and the introduction of a simplified nonlinear propagation model. The impact of the additive noise $\eta_{1,2}$ on statistical signal properties will be presented in a separate publication.

As a result of linear predispersion, the input optical signal (which we here consider to have finite initial optical bandwidth B) acquires a quadratic shift in the spectral phase. After the propagation, we apply the postprocessing loop that fully compensates both the initial predispersion and the total amount of accumulated transmission dispersion. Mathematically, the procedure can be described by introducing the *compensated* optical fields, $A_{1,2}$ in the ω domain related to the original envelope via

$$U_n(z,t) = \int_{-\infty}^{\infty} \mathrm{d}\omega e^{-\alpha z/2 + i\omega t - i\omega^2(K-\beta_2 z)/2} A_n(z,\omega). \quad (2)$$

The parameter K (measured in ps^2) is the effective accumulated dispersion introduced by the preprocessing. Then the initial field(s) (in the ω domain) is (are) given by $A_n(0,\omega)$. Applying the transform above produces the preprocessed field(s) $U_n(0,t)$, which is (are) then launched into the fiber and evolves according to Eq. (1). The postprocessing inverts mapping [Eq. (2)] at the receiver, and the effective output is the field(s) $A_n(z,\omega)$. For the new fields A_n, the master propagation equation (1) can be rewritten in an integrodifferential form [below $A_n(\omega) \equiv A_n(z,\omega)$]:

$$\frac{\partial A_n(\omega)}{\partial z} = i\frac{8\gamma}{9}\int\int \mathrm{d}\omega_1 \mathrm{d}\omega_2 e^{-\alpha z + i(\omega-\omega_1)(\omega-\omega_2)(K-\beta_2 z)} \times A_n(\omega_1)[A_n(\omega_2)A_n^*(\omega_3) + A_{3-n}(\omega_2)A_{3-n}^*(\omega_3)], \quad (3)$$

here $n = 1,2$ and $\omega_3 = \omega_1 + \omega_2 - \omega$. Effectively, the integration in Eq. (3) is restricted to the bandwidth window $|\omega| < \pi B$. Assuming that spectral oscillations due to dispersion $K - \beta_2 z$ have the smallest spectral scale (we stress that this is a rather strong requirement, and modulation and coding should be adequately adjusted to satisfy this condition), it is possible to demonstrate that the nonlinear channel can be described analytically provided that the initial precompensation K is large enough (the specific inequalities are provided below). The main idea of our approach is that as the accumulated dispersion $|\beta_2|z$ becomes large, the exponential term becomes highly oscillating and one can use a saddle-point approximation to evaluate the integral and simplify the model. However, this does not account for the initial stages of evolution when $|\beta_2|zB^2$ is not large. Introduction of a sufficient predispersion, K, at least, such that $KB^2 \gg 1$ one can use a saddle-point approximation *everywhere* along the fiber to calculate the integral in Eq. (3) (here $n = 1,2$):

$$\frac{\partial A_n(z,\omega)}{\partial z} = \frac{16}{9}\frac{i\gamma\pi e^{-\alpha z}}{K + |\beta_2|z}[|A_n|^2 + |A_{3-n}|^2]A_n. \quad (4)$$

These equations are integrable:

$$A_n(z,\omega) = A_n(0,\omega)\exp\left[\frac{16}{9}\frac{i\gamma\pi}{|\beta_2|}f\left(\alpha z, \frac{|\beta_2|z}{K}\right)I(\omega)\right], \quad (5)$$

with $I(\omega) = |A_1(0,\omega)|^2 + |A_2(0,\omega)|^2$ and $f(x,y) = e^{x/y}(Ei[-x/y - x] - Ei[-x/y])$, which in the lossless limit takes a simple form $f[0,y] = \ln(1+y)$. Here $Ei(x)$ is the exponential integral function.

Next we discuss the limitations that are imposed on the model (and amount of predispersion given by a parameter K) by considering pattern effects in the input signal. As this Letter presents a proof of concept rather than a comprehensive analysis, for the sake of clarity, in what follows rather than using a vector Manakov system, we will illustrate the key points using a close scalar nonlinear Schrödinger equation (NLSE). All the obtained results will be valid with minor modification for the vector model as well. We also assume that the effect of loss can be averaged out over periodic loss/gain evolution. Then similar saddle-point analysis in the NLSE for the compensated field yields [below $A_\omega(z) \equiv A(z,\omega)$]

$$A_\omega(z) = A_\omega(0)\exp\left\{i\frac{2\pi\gamma}{|\beta_2|}|A_\omega(0)|^2 \ln\left[1 + \frac{|\beta_2|z}{K}\right]\right\}, \quad (6)$$

which is the result to be verified numerically. Note the logarithmic dependence of the nonlinear phase shift on the distance—different from a linear dependence in the classical Gordon–Mollenauer effect [7].

The general form of the spectrum of an input sequence of M symbols sampled at a Nyquist rate $1/B$ is given by

$$A(0,\omega) = \frac{1}{B}\tilde{f}(\omega/B)\sum_{m=-M/2}^{M/2} x_m \exp\left(-im\frac{\omega}{B}\right), \quad (7)$$

where $\tilde{f}(x)$ is the dimensionless Fourier transform of a single band-limited wave form $f(\tau)$ and B is the bandwidth; for instance, it can be sinc-shaped pulses $f(\tau) = \sin(\pi\tau)/(\pi\tau)$ [8]. There are two scales in the spectrum (7). The first one relates to the FWHM of the signal waveform f. For the sinc, the spectrum is of course rectangular inside the band $|\omega| < \pi B$. The second scale is due to the pattern effect, which is given by the modulation factor in Eq. (7). One can see that the more distant symbols with large values of $|m|$ introduce shorter ω scales $\Delta\omega_{|m|} = B/|m|$. The smallest scale introduced by this factor is due to the boundary symbols $|m| = M/2$.

On the other hand, the typical scale of fast oscillations in the exponent in Eq. (3) (as well as its scalar counterpart) is $\Delta\omega_K \sim K^{-1/2}$. This means that there is a competition of the two oscillating factors in the nonlinear integral determining the spectral dynamics. In order for the saddle-point model [Eq. (5) or (6)] to work at the initial stages of pulse dynamics, the precompensation has to ensure that $\Delta\omega_K \ll \Delta\omega_f$; i.e., at least the base waveform has longer scale spectral variation than the precompensation. For all pulses adjacent to the central one and such that the inequality $\Delta\omega_K \le \Delta\omega_{|m|}$ holds (m is the symbol number counted from the central one), the saddle-point approximation [Eq. (5),(6)] works well and produces the rate of change of the spectrum $\sim(\Delta\omega_K)^{-2}$. For more distant pulses, such that the inverse inequality holds, the reciprocal regime occurs: the three-wave beating term is oscillating faster than the dispersion exponential. Such terms are not accounted for correctly by the model [Eq. (5) or (6)], but their contribution to the field equation should naturally be of a higher order than $\sim(\Delta\omega_K)^{-2}$ and hence can be neglected considering the central part of the pattern.

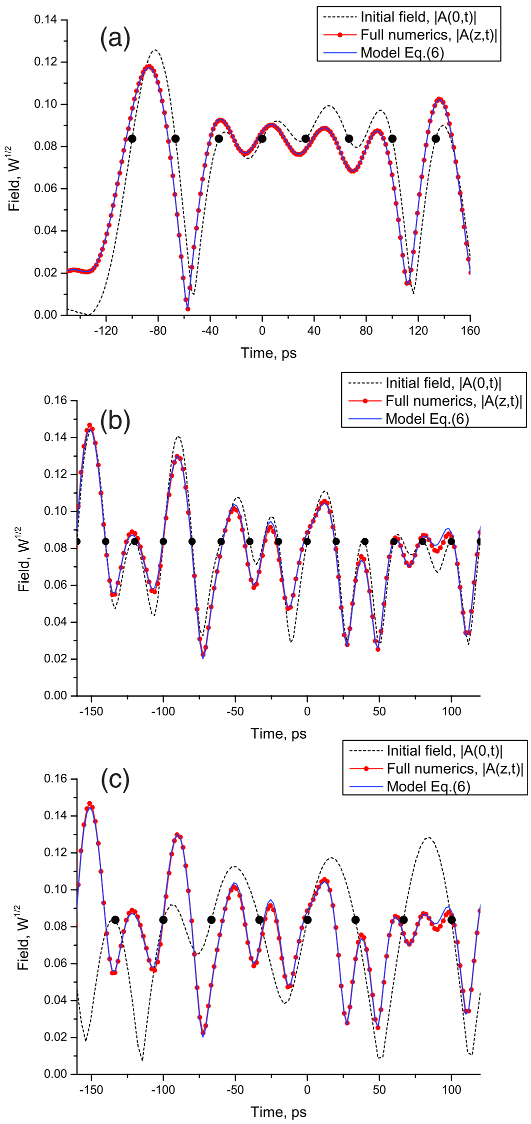

Fig. 1. Comparison of the full numerics with the saddle-point model for (a) $N = 8$ symbols, (b) $N = 32$ symbols, and (c) $N = 64$ symbols. For the latter two patterns, only a central fragment is shown. No inline dispersion compensation was used in the link.

To sum up, the saddle-point model works well if the inequality $K \gg 1/B^2$ holds. Additionally, when considering a lengthy pattern of symbols, only the symbols up to $m \sim \pm B\sqrt{K}$ contribute to the spectral dynamics—the memory effects from more distant pulses are lost.

Let us now turn to numerical verification of the proposed model {specifically, the lossless scalar case [Eq. (6)]}. Here, without loss of generality the following typical parameters are used: $\beta_2 = -20\ \mathrm{ps}^2/\mathrm{km}$, $z = 1000$ km, $\gamma P_0 = 0.021\ \mathrm{km}^{-1}$ (P_0 is the signal peak power), and $K = 2 \times 10^5\ \mathrm{ps}^2$. The results of the model [Eq. (6)] are compared with full numerical propagation in the NLSE. We use for illustration a four-level quadrature phase shift keying format (QPSK) [8] to encode a pseudorandom sequence of symbols. The results are presented in Fig. 1. Note, that although we show here results for sinc-shaped pulses, in our simulations we have also made separate runs (not shown) with Gaussian pulses of comparable width, which also demonstrated good agreement between the analytical results and the numerics. In each figure we also include for comparison the initial symbol amplitudes, $|x_m|$ (as black dots) sampled at the Nyquist rate. Because we work with the single ring PSK, all symbols have the same absolute value: $|x_m| = \sqrt{P_0}$. The bandwidth is $B = 30$ GHz (corresponding to a pulse width of $1/B = 33$ ps). Note that an increase of bandwidth B only relaxes the condition of the predispersion parameter K.

The number of symbols contributing to the saddle-point field dynamics is around $B\sqrt{K} \simeq 13$. One can see that there is no loss of accuracy of the model when the pattern length exceeds that value. Note that including a distributed optical noise into the model is straightforward [1].

In conclusion, we have proposed the application of a large predispersion (having the same sign as the transmission fiber) to an optical signal before uncompensated fiber transmission in coherent communication systems in order to simplify the following DSP. In such a nonlinear channel, the spectral dynamics of the signal can be described analytically. In the limit of very strong initial predispersion, the nonlinear propagation equations for each spectral component become local and decoupled. The relative simplicity of the mapping from input to output signal opens the possibility of fast DSP of the nonlinear effects.

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