Nonlinear inverse synthesis technique for optical links with lumped amplification

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Abstract: The nonlinear inverse synthesis (NIS) method, in which information is encoded directly onto the continuous part of the nonlinear signal spectrum, has been proposed recently as a promising digital signal processing technique for combating fiber nonlinearity impairments. However, because the NIS method is based on the integrability property of the lossless nonlinear Schrödinger equation, the original approach can only be applied directly to optical links with ideal distributed Raman amplification. In this paper, we propose and assess a modified scheme of the NIS method, which can be used effectively in standard optical links with lumped amplifiers, such as, erbium-doped fiber amplifiers (EDFAs). The proposed scheme takes into account the average effect of the fiber loss to obtain an integrable model (lossless path-averaged model) to which the NIS technique is applicable. We found that the error between lossless path-averaged and lossy models increases linearly with transmission distance and input power (measured in dBm). We numerically demonstrate the feasibility of the proposed NIS scheme in a burst mode with orthogonal frequency division multiplexing (OFDM) transmission scheme with advanced modulation formats (e.g., QPSK, 16QAM, and 64QAM), showing a performance improvement up to 3.5 dB; these results are comparable to those achievable with multi-step per span digital back-propagation.

OCIS codes: (060.2330) Fiber optics communications; (060.4370) Nonlinear optics, fibers; (070.4340) Nonlinear optical signal processing; (060.1660) Coherent communications.

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1. Introduction

Continuing demand from the growing number of bandwidth-consuming applications and on-line services (such as cloud computing, HD video streams and many others) is pushing the required communication systems capacity close to the theoretical limit of a standard single-mode fiber (SSMF) [1], which is imposed by the inherent fiber nonlinearity (Kerr effect) [2]. Various compensation techniques have been proposed in attempting to surpass the Kerr nonlinearity limit. These include digital back-propagation (DBP) [3], digital [4] and optical [5-7] phase conjugations (OPCs) at the mid-link or installed at the transmitter [8], and phase-conjugated twin waves [9-12]. However, many challenges and limitations remain in applying these nonlinear compensation techniques, because the transmission technologies utilized in optical fiber communication systems were originally developed for linear (radio or open space) communication channels. Therefore, it would constitute a significant development in fiber-optic systems if the fiber nonlinearity could be taken into account in a "constructive way" when designing the core optical communication coding, transmission, detection, and signal processing approaches.

The propagation of optical signals in the SSMF can be quite accurately modelled by the nonlinear Schrödinger equation (NLSE) [13], which describes the continuous interplay between effects of fiber dispersion and nonlinearity. It is well known that the lossless (non-perturbed) NLSE belongs to the class of the so-called *integrable nonlinear systems* [14-18], which make it possible to present the field evolution in the NLSE channel within a special basis of *nonlinear normal modes*, including non-dispersive soliton modes and quasi-linear dispersive radiation. The evolution of such special nonlinear modes is essentially linear, which means that the nonlinearity-induced cross-talk between these modes is effectively absent during the propagation (in an idealized system without signal corruption due to noise) [14-26]. As a result, these nonlinear modes can potentially be used to encode information that, in turn, can be recovered at the receiver without suffering from nonlinear impairments [15, 19-23]. This general idea was first introduced by Hasegawa and Nyu in [19], and termed there as "eigenvalue communications": In this prefiguring approach the solitonic eigenvalues were used, and the authors of Ref.[19] utilized the invariance of the discrete eigenvalues (that is, those attributed to the solitonic degrees of freedom) of the Lax operator associated with the NLSE.

Over the recent years, several groups have revisited and extended the original ideas of Hasegava and Nyu, in the context of coherent optical communications. The concept of eigenvalue communications itself is being now approached from two somewhat "orthogonal and complimentary" pathways, neither of which excludes the parallel implementation of the alternative approach. These two main directions in the eigenvalue communications methodology can be categorized according to what part of the nonlinear spectrum (solitonic discrete part or continuous part, corresponding to nonlinear radiation) is used for the modulation and transmission. In particular, [19-22, 24, 25] studied the discrete (solitonic) components of the nonlinear spectrum for data communications. This approach requires considerable optimization of the pulse shapes (solitonic alphabet) for the purpose of maximizing the resulting spectral efficiency (SE). The modulation of the continuous part of the nonlinear spectrum has also been considered in [22, 25] for low power region, where the solitonic part does not exist. However, no scheme for encoding the transmitted information onto the continuous spectrum was proposed. Thus, highly complex maximum likelihood detection scheme has to be applied at the receiver, making the system unsuitable for practical implementations.

Recently, an efficient approach for achieving a high SE transmission, based on the modulation of the continuous part of the nonlinear spectrum, has been proposed in [26] and assessed by the authors (for the Raman amplification) in [27] – the nonlinear inverse synthesis (NIS) method. In this technique, the encoded input signal (or, to be more specific, its linear Fourier spectrum) is mapped onto a complex field in the time domain via the

Gelfand–Levitan–Marchenko equation (GLME) before transmission. An complex optical temporal signal to be launched into fiber characteristic for NIS can be created by modern arbitrary waveform generators. This step makes it possible to translate directly a standard modulation format into the nonlinear spectral domain and avoid the difficulties associated with the discrete nonlinear spectrum.

In addition to the high performance offered, the NIS method is also highly flexible as it can be integrated with different modulation formats and transmission schemes [27], so that we use the OFDM modulation of the nonlinear spectral data as a straightforward example. It was shown in [27] that NIS can provide a performance gain up to 4.5dB, which is comparable with multistep per span digital back propagation (DBP). However, an important remaining shortcoming of the NIS scheme is that it is suitable only for lossless optical links, such as those with distributed Raman amplification.

In this paper, we make an important extension of the NIS technique and the entire nonlinear Fourier transform (NFT) method to optical links with EDFA-based amplification, by applying the lossless path-averaged (LPA) NLSE model [14, 28, 29]. We demonstrate how the LPA NLSE approach can be used to develop an appropriate NIS scheme for optical links with EDFA-based amplification, offering a similar performance gain to those achieved with the NIS scheme for lossless optical links. Using numerical simulation, we demonstrate the feasibility of this approach with high SE transmissions. For the sake of comparing the performance with the results of Ref. [27], we consider here 112 Gb/s, 224 Gb/s, and 336 Gb/s orthogonal frequency division multiplexing (OFDM) coherent transmission systems with QPSK, 16QAM, and 64QAM modulation formats. To show the potential benefit of this approach, we also compare the performance of the NIS system with the multi-step per span DBP. Our Monte Carlo simulations show that, with a proper design, the high-SE NIS-based communication system with EDFA-based amplification can offer a performance improvement of up to 3.5 dB, which is comparable with the results achievable with the multi-step per span DBP.

2. LPA NLSE for optical links with EDFA-based amplification

In this section, we remind the derivation of the LPA NLSE for modeling the propagation of signal in optical links with EDFA-based lumped amplification [14, 28, 29]. We start with the standard NLSE governing the propagation of a complex slow-varying optical-field envelope $q(z, t)$ along a single-mode optical fiber [14, 28, 29] (that is, in-between two consecutive amplifiers):

$$jq_z - \frac{\beta_2}{2} q_{tt} + \gamma q |q|^2 = -j\frac{\alpha}{2} q, \tag{1}$$

where z stands for the propagation distance and t is the retarded time in the frame co-moving with the group velocity of the envelope. Here, we focus on the case of anomalous dispersion (that is, the constant chromatic dispersion coefficient is $\beta_2 < 0$ in Eq. (1)) and hence deal with the so-called focusing type of NLSE [15-18]. The higher-order dispersion terms are not considered here; γ is the instantaneous Kerr nonlinearity coefficient, and α is the fiber loss coefficient. By introducing the standard [14, 28] change of variable:

$$q = \exp(-\frac{\alpha}{2} z) A, \tag{2}$$

Eq. (1) can be written as:

$$jA_z - \frac{\beta_2}{2} A_{tt} + \gamma \exp(-\alpha z) A |A|^2 = 0 \tag{3}$$

Assuming (following Refs.[14, 28]) that the dynamic of the envelope A(z, t) does not change significantly after each fiber span having the length L, the distance-dependent nonlinear coefficient in (3) can be replaced by its average value over each fiber span:

$$\gamma_1 = \frac{1}{L}\int_0^L \gamma \exp(-\alpha z) dz = \gamma \frac{G-1}{G\ln(G)}, \quad (4)$$

where G is total loss over the fiber span, $G = \exp(\alpha L)$. In other words, this is a well-known replacement of space-varying nonlinear phase shift by the average nonlinear phase shift (an effective nonlinear length). The obtained LPA NLSE can be written as:

$$jA_z - \frac{\beta_2}{2} A_{tt} + \gamma_1 A|A|^2 = 0 \quad (5)$$

Given the input field q(z,t), the normalized mean square error (NMSE) produced by replacing the exact model (1) with the LPA NLSE (5), is introduced as:

$$NMSE = \frac{\left\langle |q_2(Z,t) - q_1(Z,t)|^2 \right\rangle}{\left\langle |q_1(Z,t)|^2 \right\rangle}, \quad (6)$$

where <·> stands for the averaging over the whole considered time interval, q_1(Z, t) and q_2(Z, t) are the two output fields obtained using the standard NLSE (1) and the LPA NLSE model (5).

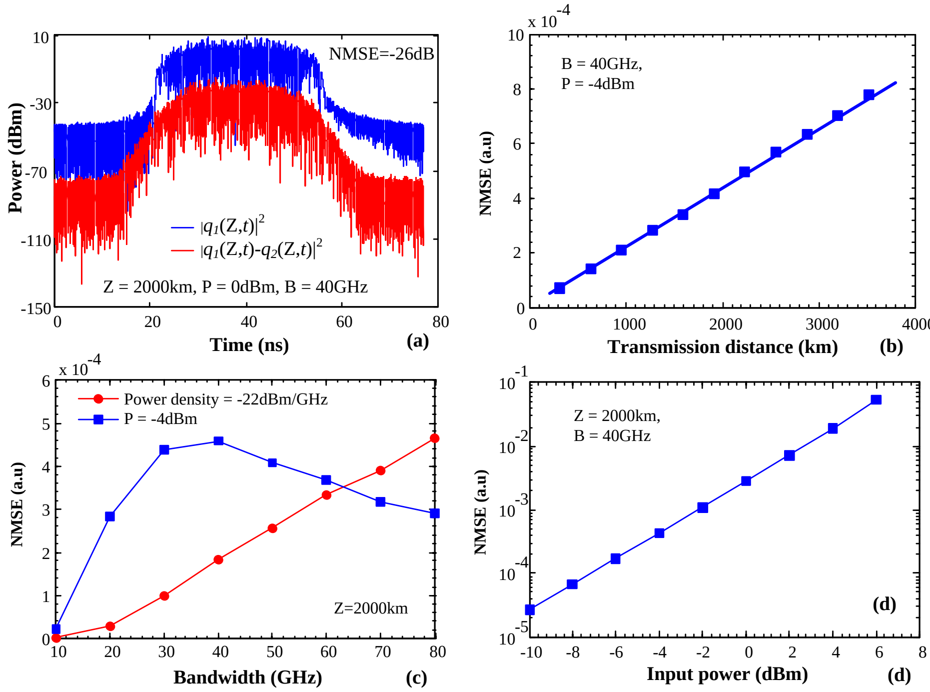

Fig. 1(a) – A comparison of output fields obtained by using the standard NLSE and the LPA NLSE, the amplifier spacing is 80km. (b) – NMSE as a function of the transmission distance. (c) – NMSE as a function of the signal's bandwidth for a given input power and a given input power density. (d) – NMSE as a function of the input power.

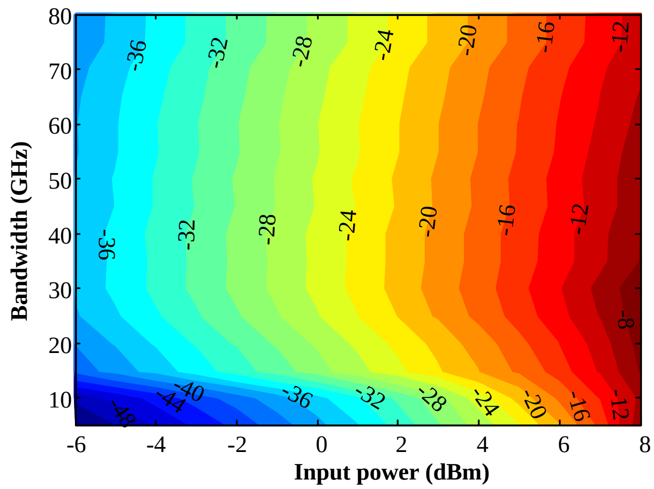

Fig. 2. Level curves of NMSE (in dB) indicating the error of using LPA NLSE (5), plotted as a function of the signal's bandwidth (in GHz) and the input power (in dBm) for the propagation distance 2000km.

The NMSE can be considered as the effective inverse signal-to-noise ratio (SNR), indicating the relative noise power introduced by the inaccuracy of the LPA NLSE model. As a result, we can expect that a digital signal processing (DSP) technique based on the usage of the model (5) would contribute some additional effective noise to the processed signal. The power of this additional noise can be estimated using the NMSE calculated by (6). Taking into account the fact that the current optical communication systems employing forward error correction (FEC) are designed to deal with a received SNR of less than 10 dB, we can predict that a NMSE of less than -20 dB (<0.01) would not have a significant impact on the effectiveness of the DSP based on the model (5). As a result, we believe that the model (5) can be used if the resulted NMSE is less than -20 dB.

Herein, we numerically study the validity of the model (5) for the optical links with the EDFA-based amplification by considering the NMSE and its dependence on critical parameters as such the signal's bandwidth, input power and the link distance. As an example, we take into account a QPSK Nyquist-shaped system. The modulation format of choice is not critical here. The input field consists of 2^{10} QPSK symbols with sinc pulse-shape. The optical links consist of 80 km spans of SSMF with a loss parameter of 0.2 dB/km, nonlinearity coefficient of 1.22 /W/km, and dispersion coefficient 16 ps/nm/km. In Fig. 1(a) the mismatch between the output fields obtained by the standard NLSE and the LPA NLSE is plotted. The corresponding *NMSE* is 0.0024 (~ - 26 dB), which indicates that the LPA NLSE can be used effectively to model the propagation of signal in optical links with lumped amplification. However, as the LPA NLSE is an approximate approach, its accuracy depends on the system and signal parameters. Fig. 1(b) shows that the NMSE increases linearly with the transmission distance. In addition, as can be seen in Fig. 1(c), the NMSE also increases almost linearly with the signal bandwidth when the power spectral density is fixed. If the input power is fixed, the NMSE increases with the signal's bandwidth only when the bandwidth is small. With bandwidth larger than approximately 40 GHz, the NMSE decreases with the increase of the signal's bandwidth. This phenomenon can be understood if we recall that when the bandwidth is increased while the signal power is fixed, the power spectral density decreases effectively reducing the impact of fiber nonlinearity. This, in turn, reduces the impact of the distance-dependent nonlinear coefficient in (3). Fig. 1(d) shows that the NMSE increases linearly in the log-scale with the input power (increase with P^k, where k~2), showing that the input power is the most critical parameter in applying the LPA NLSE to model the propagation of signal in optical links with EDFA-based amplification.

Figure 2 shows the NMSE as a function of the signal's bandwidth (in GHz) and the input power (in dBm) for a 2000km link. Taking a value of NMSE of -20 dB as the threshold, Fig. 2 reveals that the LPA NLSE can be used to model the propagation of signal in optical links with EDFA-based amplification if the launch power is less than 3dBm, almost independently of the signal bandwidth. In addition, the launch power threshold can be increased by increasing the signal bandwidth. This leads to an important result that an appropriate NIS scheme based on the model (5) can be used effectively to combat the fiber nonlinearity impairments in optical links with the EDFA-based amplification.

3. NIS for optical links with lumped amplification

The NIS method for optical links with lumped amplification includes three basic steps, as shown in Fig. 3(a). At the first step, which is performed at the transmitter side, the linear Fourier spectrum of the encoded complex waveform ($S(\omega)$) is mapped onto the continuous part of the nonlinear spectrum of a complex signal q(t) by solving the GLME (the backward nonlinear Fourier transform (BNFT)). The mapping operation of the BNFT block can be expressed as:

$$r(\xi)\Big|_{\xi=-\omega/2} = -S(\omega) \tag{7}$$

At the second step, the continuous part of the nonlinear Fourier spectrum of the received signal is obtained at the receiver by solving the Zakharov–Shabat problem (ZSP or the forward NFT (FNFT)). At the final step, a single-tap dispersion removal operation is performed to remove all deterministic nonlinear impairment as:

$$\overline{S}(\omega) = -r(L,\xi)\cdot e^{2j\xi^2 L}\Big|_{\xi=-\omega/2} \tag{8}$$

After dispersion removal operation, IFFT operation is performed to reconstruct the initial encoded complex waveform, which is then fed to the standard OFDM receiver. The numerical procedures for solving the GLME and ZSP can be found in [26, 27]. Both the algorithms employed to solve the GLME and ZSP in this work require O(D^2) floating point operations where D is the number of samples.

In general, the only difference between the NISE scheme for the lossless optical links and the links with EDFA-based amplification is related to the normalization procedure, as the GLME and ZSP are appropriate only for the NLSE in the normalized form. Before solving the GLME, the input optical field $s(t)$ is normalized using the LPA NLSE (5) as follows:

$$\frac{t}{T_s} \to t, \quad \frac{z}{Z_s} \to z, \quad s\sqrt{\gamma_1 Z_s} \to s \tag{9}$$

where T_s is a free time normalization parameter (e.g., a characteristic time scale of the input waveform) and the associated space scale is $Z_s=T_s^2/\ \beta_2$; γ_1 is the path-averaged nonlinear coefficient defined by Eq. (4).

After normalization (9), the input signal is fed into the BNFT, which maps the linear Fourier spectrum of this input signal to the continuous part of the NFT of a complex signal q(t). De-normalization, which is the inverse procedure of (9), is then applied to q(t), to generate the signal for feeding it into the IQ modulator. At the receiver, after coherent detection, normalization procedure (7) is performed before solving the ZSP.

As discussed in [27] the FNFT and BNFT are performed with the signals decaying to zero as boundary conditions ($t \to \pm\infty$), the proposed NIS approach for links with EDFA-based amplification is appropriate for the burst mode transmission (Fig. 3(b)) of a multi-access network, in which neighboring packet data are separated by a guard time. The guard time duration is usually chosen to be longer than the channel memory, which in our case is the fiber chromatic dispersion-induced memory. Different packet data can be sent from the same or different transmitters. In this work, we assume for the sake of simplicity that all packet data are from the same transmitter. The more general case of periodic boundary conditions will be considered in our future publications.

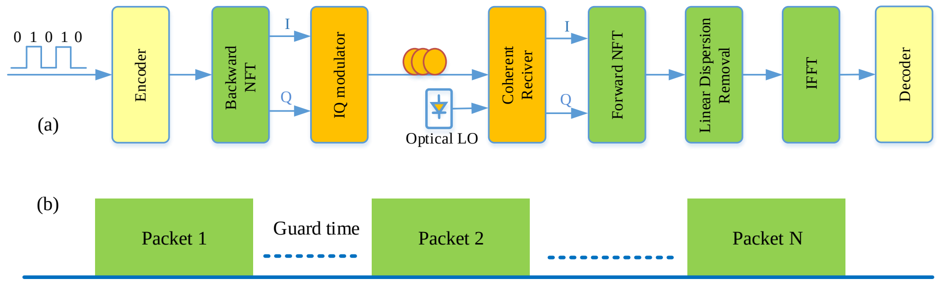

Fig. 3(a) – Block diagram of NIS-based optical communication systems. (b) – Illustration of a burst mode transmission, in which neighbouring packet data are separated by a guard time.

4. Simulation results and discussion

In order to investigate the performance of the proposed NIS approach for optical links with EDFA-based amplification, we take into account the OFDM transmission scheme because of the high SE provided [30, 31]. A comparison of the OFDM modulation with a single carrier using Nyquist pulse shaping (or orthogonal time division multiplexing) for NIS-based systems was provided in [27], revealing that the OFDM is a more suitable modulation format because it provides a smaller $L1$-norm. This can be explained by the fact that for conventional signals, the L1 norm of the Fourier transform is always lower or equal to the L1 norm of the time-domain signal. Therefore, in this paper we consider only the OFDM scheme. We designed 56-Gbaud OFDM NIS-based transmission systems (in burst mode) with high SE-modulation formats; namely, QPSK, 16QAM, and 64QAM. The net data rates of these systems, after removing 7% overhead due to the FEC, were 100 Gb/s, 200 Gb/s, and 300 Gb/s (considering only the burst's bit-rate), respectively. Herein, we aim to show that DSP techniques based on model (5) can be applied effectively even in the long-haul optical communication systems with as large a bandwidth as 56 GHz.

For the OFDM NIS-based system, the IFFT size was 128, where 112 subcarriers were filled with data (with Gray-coding) while the remaining subcarriers (6 subcarriers) were set to zero. The useful OFDM symbol duration was 2 ns and no cyclic prefix was used for the linear dispersion compensation. The oversampling factor was 40, leading to a sampling rate of 2.24THz. After the IFFT, the time domain OFDM signal was normalized following (9) and then fed into the BNFT block. For simplicity, here we assume that each packet data contains only one OFDM symbol. In addition, only one packet generated with random data was transmitted and the system performance was evaluated through Monte Carlo simulation with 100 runs. The optical link is assumed to consist of 80 km SSMF. The noise figure of EFDA was 5dB. In simulation the noise was added distributedly after each fiber span to capture correctly the nonlinear signal-noise interaction.

The guard time duration is chosen to be 20% longer than the fiber chromatic dispersion induced memory for a 2000km-link. The dispersion induced memory of the link can be estimated as:

$$\Delta T = 2\pi B\beta_2 L, \qquad (9)$$

where B is the signal's bandwidth and L is the link distance. In our case, by considering a bandwidth of 56 GHz, a transmission distance of 2000km we have $\Delta T \approx 14$ ns. In the simulation the guard time duration in each packet was chosen to be 17 ns, leading to a packet duration of 19 ns (including both guard time duration and OFDM symbol).

4.1. NIS performance without the ASE noise

It has been shown in [27] that when the ASE noise is ignored (fiber nonlinearity is the only system's impairment) the NIS method can perfectly compensate the deterministic impairment due to the fiber nonlinearity, using just a single-tap linear dispersion removal for the nonlinear spectrum at the receiver. In other words, the fiber nonlinearity has no impact on the system performance, which is now limited by the transceiver's impairments. It has been also noticed in [27] that the errors introduced by the numerical BNFT and FNFT operations are the only limiting factors in a NIS-based system when the ASE noise is ignored. On the other hand, as the NIS method for optical links with the EDFA-based amplification is developed from the approximate LPA NLSE, the perfect nonlinearity compensation cannot be achieved even in the absence of the ASE noise.

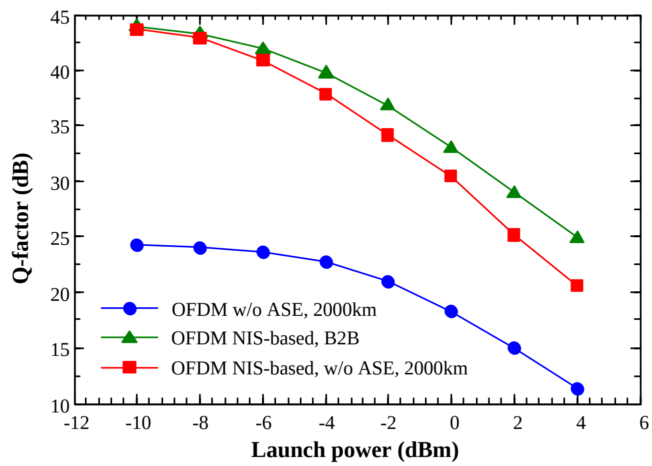

Fig. 4. Q-factor as a function of the launch power for 100 Gb/s QPSK OFDM NIS-based system in the back-to-back case and in a 2000 km optical link, the ASE is ignored.

A comparison of back-to-back performance and the transmission performance when ignoring the ASE noise would indicate the performance penalty associated with the use of PLA NLSE in the NIS scheme for links with EDFA-based amplification. Such comparison is shown in Fig. 4 for 112 Gb/s QPSK OFDM NIS-based system in a 2000 km optical link. The Q-penalty associated with the use of PLA NLSE increases with the launch power. At a launch power of 4 dBm, the PLA NLSE would give a Q-penalty of 5 dB. This phenomenon can be explained by the fact that the inaccuracy (measured with the NMSE) of the PLA NLSE increases with the input power (Fig. 1(d)). As a result, any DSP technique based on the PLA NLSE would potentially provide a performance penalty, which also increases with the input power. In addition, it was shown in [27] that the accuracy of the numerical algorithms employed here for solving the GLME and ZSP problems also decreases when signal's power is increased. As a result, the performances of NIS-based systems are limited at high power by the numerical errors.

4.2. Performance comparison of NIS versus DBP in the presence of ASE noise

In this subsection, in order to demonstrate the feasibility of the proposed NIS method for practical optical links with EDFA-based amplification, we compare the performance of the NIS with DBP in high-SE OFDM transmission systems. For the implementation of DBP, the received signal is first filtered with an 8^{th}-order low-pass filter with a bandwidth of 40 GHz. Subsequently, the optical field is reconstructed and the signal is back-propagated with a different number of steps per single span. The performances for QPSK and 16QAM systems were evaluated using the well-known error vector magnitude (EVM), while direct error counting was adopted for 64QAM. The measured BER for the discussion convenience is then converted to an equivalent "Gaussian noise" Q-factor [32] in dB for our convenience in discussion.

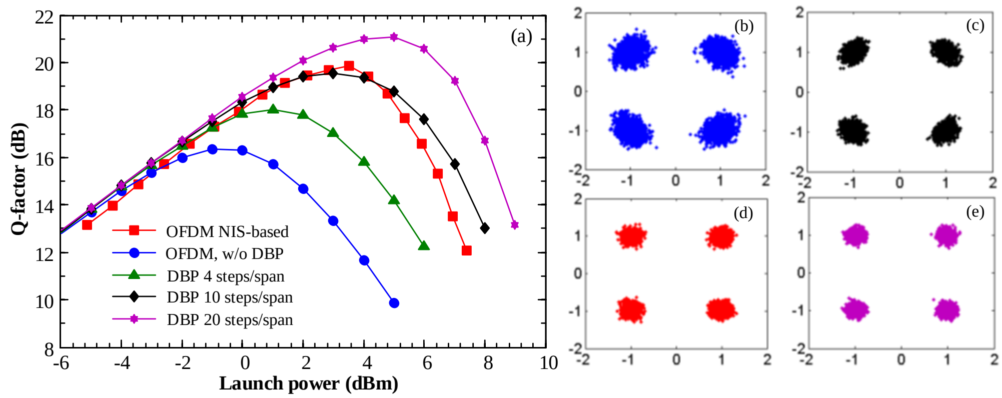

Fig. 5(a) – Performance comparison of the 100-Gb/s QPSK OFDM systems with the NIS vs. the DBP methods for fiber nonlinearity compensation, and constellation diagrams at the optimum launch powers for the cases: (b) – without NIS and DBP, (c) DBP with 10 steps/span (d) with the NIS method, (e) DBP with 20 steps/span. The propagation distance is 2000km.

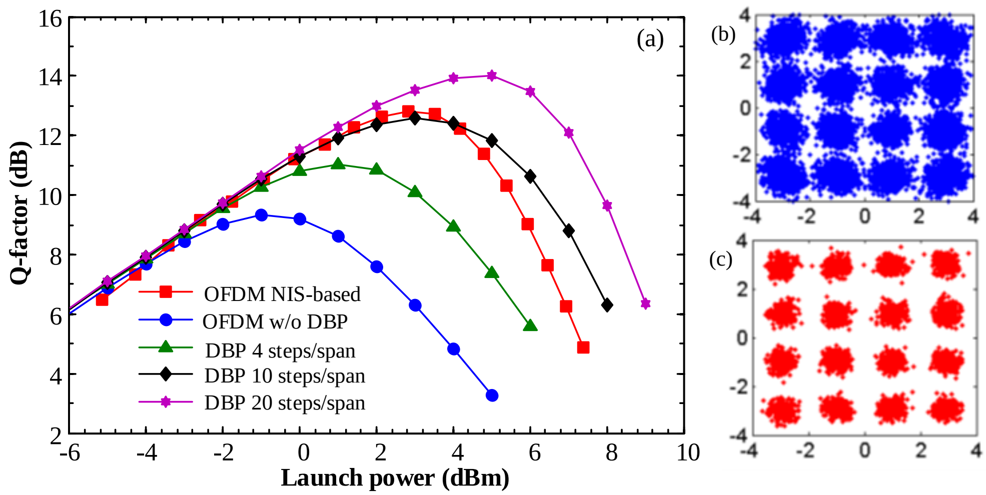

Fig. 6. (a) – Performance comparison of the 200-Gb/s 16QAM OFDM systems with the NIS vs. the DBP methods for fiber nonlinearity compensation and constellation diagrams at the optimum launch powers for (b) – Without NIS and DBP, (b) with the NIS method. The transmission distance is 2000km.

The comparison of NIS and DBP for OFDM systems with QPSK, 16QAM and 64QAM modulation formats are shown in Fig. 5, Fig. 6, and Fig. 7 respectively. It can be seen that, almost independently of modulation formats, the proposed NIS method offers a performance gain of approximately 3.5 dB over the OFDM system without DBP. This performance gain is comparable with those achieved with a highly complex DBP with 10 steps per span. The results obtained here agree well with prior results presented in [22] for low signal region, where the modulation of the continuous part can be directly achieved as the discrete part of the nonlinear spectrum does not exist. However, beside the serious limitation in the signal power, the approach proposed in [22] also requires highly complex maximum likelihood detection scheme, making challenging its implementation for practical applications.

It should be mentioned that the NIS method for optical links with EDFA-based amplification proposed here provides the same complexity as the NIS method proposed for lossless optical links. Readers who are interested in an estimation of the complexity of NIS method in a comparison with DBP are referred to [27]. We believe that the overall complexity of the NIS method could be significantly reduced further by taking into account recent advancement in fast algorithms for performing the BNFT [33-35] and FNFT [34, 36, 37] and thus can be even lower than DBP employing more efficient algorithms [38].

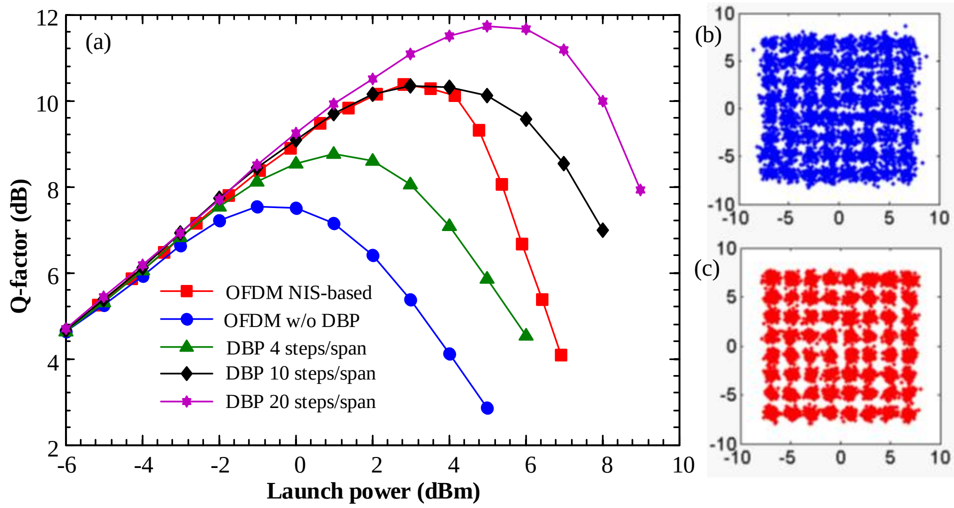

Fig. 7(a) – Performance comparison of the 300-Gb/s 64QAM OFDM systems with the NIS vs. the DBP methods for fiber nonlinearity compensation and constellation diagrams at the optimum launch powers for (b) – Without NIS and DBP, (b) with the NIS method. The transmission distance is 640km.

Figures 5 to 7 clearly demonstrate the high performance of the proposed NIS scheme for optical links with lumped amplification. However, the performance of the NIS reaches an optimum value and then degrades as the launch power increases. The reasons for this decrease are the numerical errors associated with the BNFT, FNFT and the inaccuracy of the LPA NLSE. The impact of all these different limiting factors grows with the launch power. As discussed in [27], the numerical errors associated with the BNFT, FNFT can be reduced by increasing the sampling frequency or by developing more accurate BNFT and FNFT, even though both approaches are very practically challenging. On the other hand, the additional noise associated with the inaccuracy of the LPA NLSE is fundamental and, in general, cannot be avoided. This type of error limits the performance of a NIS-based system at a high power level. In Figs. 5–7, it can be seen that the system performance degrades faster in the high power region when the constellation size is increased. We attribute this phenomenon to the non-Gaussian statistics of the numerical errors associated with PLA NLSE model, which affects different modulation formats unequally. In addition, Fig. 12 from [27] demonstrates that the noise tolerance of the NIS scheme can actually be higher than that of the DBP algorithm: the Q-factor curve for the NIS method in the noise-dominated regime (low signal powers) lies above that for the DBP.

5. Conclusion

In this work, we have proposed an extension of the NIS scheme and of the entire nonlinear Fourier transform approach based on the usage of path-averaged NLSE, which can be effectively applied in optical links with EDFA-based amplification. The Q-factor penalty associated with the usage of path-average model depends strongly on systems' parameters such as power, distance and bandwidth. The performance of the proposed NIS scheme was numerically investigated in OFDM-based systems with high-SE modulation formats – namely QPSK, 16QAM and 64QAM – showing a performance gain of approximately 3.5 dB. This is comparable with the values achieved with a highly complex DBP scheme employing 10 steps per span. This investigation reveals the high potential of NIS as an alternative nonlinear compensation technique, which is effective in both lossless and EDFA-based optical links.

Acknowledgment

The support under the UK EPSRC Program Grant UNLOC (EP/J017582/1) is gratefully acknowledged.