

On Optimal Modulation and FEC Overhead for Future Optical Networks

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Abstract: Transceivers employing square PM-QAM with up to two FEC overheads are optimized based on the SNR distribution. For NSFNET two FEC overheads with PM-16QAM give an 82% throughput increase compared with PM-QPSK with 7% overhead.

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1. Introduction and Motivation

Future optical networks will use polarization-multiplexed (PM) multilevel modulations and forward error correction (FEC). This combination is known as coded modulation (CM) and its design requires the joint optimization of the FEC encoder and modulation format. The optimum receiver structure for CM is the maximum likelihood (ML) receiver, which finds the most likely *coded sequence*. The ML solution is in general impractical, and thus, very often the receiver is implemented as a (suboptimal) bit-wise (BW) receiver instead. In a BW receiver, hard or soft information on the code bits is calculated first, and then, an FEC decoder is used (see Fig. 1). Although BW receivers are very common for soft-decision FEC, for simplicity, in this paper we consider CM with hard-decision FEC (HD-FEC) only.

Most of the analyses of optical networks assume a fixed FEC overhead (OH) [1,2]. However, variable OHs and/or modulation sizes have been recently considered in, e.g., [3–6]. From a theoretical point of view, fixing the OH is an artificial constraint that reduces flexibility of the CM design. Fixed OHs also decrease the overall network throughput.

In this paper, we study the problem of finding optimal modulation sizes and FEC OHs from an information theory viewpoint. For point-to-point links, the solution depends only on the signal-to-noise ratio (SNR). For an optical network, the solution depends on the SNR *distribution* of the connections. The theoretically optimum CM design is obtained when the modulation size and FEC OH are *jointly* designed. When applied to NSFNET as a reference topology, significant increases in network throughput are shown, even when realizable structures are considered.

2. System Model

We consider a two-polarization, discrete-time, memoryless, additive white Gaussian noise (AWGN) channel. The components of the noise $\underline{Z}$ are independent, zero-mean, Gaussian random variables with total variance $N_0/2$ per polarization. This channel model (also known as the GN model [7]) characterizes optically-amplified links dominated by amplified spontaneous emission noise, and treats the power-dependent nonlinearities in the presence of sufficient dispersion as an additional source of AWGN.

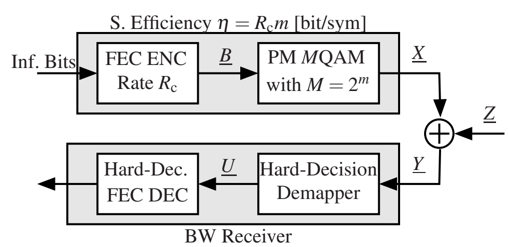

Fig. 1. CM transceiver structure.

At each discrete-time instant, the transmitted symbols in each polarization $\underline{X}$ are selected with equal probability from a discrete constellation with $M = 2^m$ constellation points (see Fig. 1). We consider square PM-QAM constellations with $m = 2,4,6,8,10,12$ (i.e., MQAM with $M \leq 4096$) labeled by the binary-reflected Gray code. For a rate R_c FEC encoder which maps the *information bits* into code bits $\underline{B}$, the spectral efficiency (SE) is $\eta = 2R_c m$ [bit/sym]. At the receiver side an HD demapper is used. The estimated bits $\underline{U}$ are then passed to an HD-FEC decoder which gives an estimate of the information sequence.

3. Optimal Modulation and FEC Overhead

The capacity of the AWGN channel is $C = 2\log_2(1+E_s/N_0)$ [bit/sym], where E_s is the average symbol energy and E_s/N_0 is the SNR. The value of C represents the maximum number of information bits per symbol that can be reliably transmitted through an AWGN channel. In practice, however, the modulation has M discrete levels. In this case, from an information theory point of view, the optimal rate and constellation size can be chosen from the mutual information (MI). In particular, for a given constellation, the rate of the encoder should fulfill $R_c \leq I(X;Y)/(2m)$, where $I(X;Y)$ is the MI between the input and output of the channel. The FEC OH can then be calculated as $\text{OH} = 100(R_c^{-1} - 1)\%$. The MI curves indicates that, regardless of the SNR, and in order to maximize the SE η, the densest available constellation should always be used and the code rate chosen between 0 to 1.[1] This has been shown, .e.g., in [5, Fig. 1], [7, Fig. 11].

The symbol-wise MI $I(X;Y)$, however, *is not* a relevant quantity for an HD-BW receiver like the one in Fig. 1. Reliable transmission for an HD-BW receiver and a given constellation is possible if $R_c \leq I(B;U)$, where $I(B;U) \triangleq (1 - H_b(\text{BER}))$ [6, eq. (5)] is a *bit-wise* MI and $H_b(\cdot)$ is the binary entropy function. Here, BER is the average pre-FEC bit-error rate, and B and U are the binary random variables in $\underline{B}$ and $\underline{U}$ (see Fig. 1), resp.

The values of $I(B;U)$ for $M = 4, 16, \ldots, 4096$ are shown in Fig. 2, where the BER was calculated using [8, Theo. 2]. The key difference between the achievable rates $I(B;U)$ and $I(X;Y)$ is that the former cross each other for different values of M (see the squares in Fig. 2). This has been shown in [6, Fig. 1] and it also happens if a soft-decision BW receiver is considered (see [9, Fig. 4]). The most important consequence of crossing achievable rate curves is that the theoretically optimal choice of R_c and M is not straightforward. For the BW with HD-FEC we consider here, QPSK should be used for SNRs below $E_s/N_0 \leq 5.8$ dB, 16QAM for $5.8 \leq E_s/N_0 \leq 14$ dB, etc. Nevertheless, for a point-to-point link with a given SNR, the optimal parameters can be directly obtained from Fig. 2.

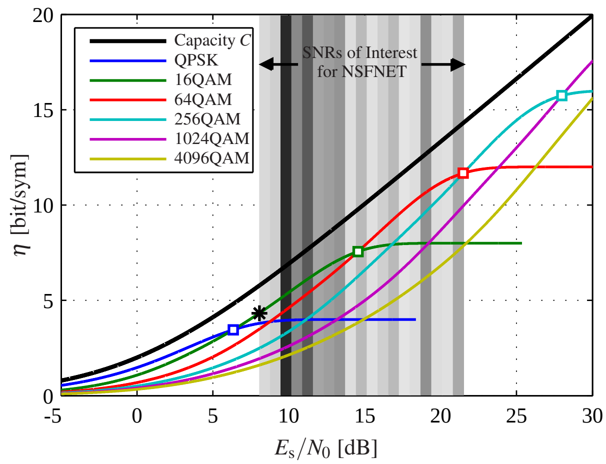

Fig. 2. Achievable rates for BW receiver with HD-FEC.

In Fig. 2 we also show the SNR values of interest for the NFSNET we study in Sec. 4, where the darkness of the stripes id proportional to the probability of observing connections with that particular SNR. The minimum and maximum SNRs are 8 dB and 21.5 dB, resp., which correspond to 16QAM with $R_c = 0.54$ (shown with an asterisk in Fig. 2) and 64QAM with $R_c = 0.97$. These modulation and coding rates correspond to OHs of 85% and 3%, resp.

4. Modulation and FEC Overhead in NFSNET

We consider the reference 14 node, 21 link NSF mesh topology (see [2, Fig. 7]) and solve the routing problem as described in [2]. In particular, we maximize the network throughput under uniform demand, based on the capacity function C for each connection. The SNR of each connection was calculated assuming EDFAs with 5 dB noise figure, 80 km spans, and the incoherent GN model of nonlinear interference [7]. The interference was taken as that on the worst case central DWDM channel, i.e., we assumed that all the links were fully loaded with 80 DWDM channels of $R_s = 32$ GBaud Nyquist *sinc* pulses on a 50 GHz fixed grid with an equal and globally optimal launch power. SPM was assumed to be ideally compensated and the ROADM nodes were assumed lossless.

The results of the routing optimization gave a network throughput of 279 Tbps and utilized 1100 connections. This network throughput is the maximum achievable throughput for the uniform demand matrix and is calculated as the product of the number of node pairs and the minimum throughput between any node pair. The SNR distribution across 1100 connections is schematically shown in Fig. 2. Using the obtained 1100 SNR values, we computed the throughput obtained by considering variable-rates and variable-constellations. In this case, all the transmitters are assumed to be able to continuously adjust R_c and also ideally choose the constellation size (using the results in Fig. 2). The resultant throughput is 218 Tbps and is shown in Fig. 3 (left) together with the capacity-based throughput. Due to the continuous-rate and/or continuous-constellation assumptions, these two throughputs are upper bounds that cannot be achieved in practice. On the other extreme, the simplest realizable alternative is to use QPSK with a fixed OH of 7%. This gives a throughput of 86.7 Tbps (see Fig. 3 (left)).

To improve upon QPSK with 7% OH, we consider a scheme where all the transmitters use the same constellation size M and the same code rate, but where the code rate is optimized based on the achievable rates in Fig. 2. The obtained throughputs are shown in Fig. 3 (left) and show a maximum of 117 Tbps obtained with 16QAM and 61% of OH (see Fig. 3 (right)). In this case, the optimal OHs increase as M increases, however, using constellations larger

[1] In practice, of course, this is difficult as using very dense constellations at low SNRs are hard to realize.

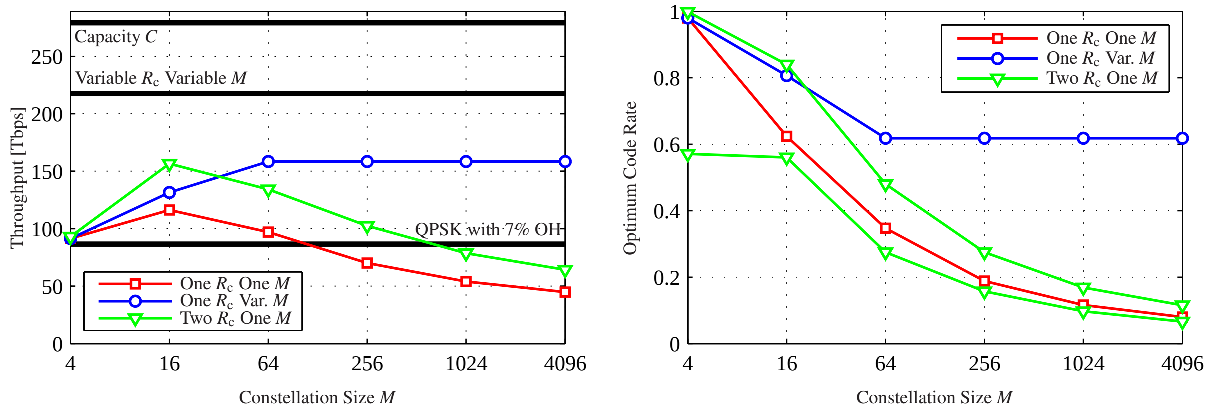

Fig. 3. Throughput (left) and optimum code rate (right) for different modulation and coding schemes.

than 16QAM reduces the throughput. This is a consequence of the SNR distribution shown in Fig. 2.

We also consider a one-rate variable-constellation scheme, i.e., where all transmitters use the same OH but different constellation sizes. The obtained results are shown in Fig. 3 (with blue lines), where the horizontal axes should be understood as the *largest* available constellation. These results show that using only 3 constellations ($M = 4, 16, 64$) and a code rate $R_c = 0.62$ is enough to harvest all the gains offered by this scheme (≈ 158 Tbps). The gains with respect to QPSK and 7% OH are in this case 82%.

Lastly, we consider a two-rate one-constellation scheme, where all transmitters use the same constellation and two optimized OHs. The results are shown in Fig. 3 (left) and indicate that using 16QAM and two OHs give approximately the same throughput obtained by using one OH and 3 modulation formats. For 16QAM, the optimum code rates are $R_c = 0.56$ and $R_c = 0.84$ (see green triangles in Fig. 3 (right)), where the first one is in fact very similar to the one obtained by considering 16QAM and the smallest value in the SNR distribution (shown with an asterisk in Fig. 2). The intuition behind this is that when the transmitters are equipped with two OHs, one low code rate can be used for the worst performing connection, while the other rate is used to increase the overall network throughput. Similar results are expected when soft-decision FEC and other network topologies are considered.

5. Conclusions

Optimal constellation sizes and FEC overheads for optical networks were studied. Joint optimization of the constellation and FEC OHs was shown to yield large gains in terms of overall network throughput. The optimal values were shown to be dependent on the SNR *distributions* within a network. Using NSFNET as a reference topology, 16QAM and two code rates gave an good throughput-complexity tradeoff. Denser constellations are expected to be more relevant in networks with higher SNRs.

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