Predicting the Performance of Nonbinary Forward Error Correction in Optical Transmission Experiments

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Abstract: It is shown that the correct metric to predict the performance of coded modulation based on nonbinary FEC is the mutual information. The accuracy of the prediction is verified in an optical experiment.

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1. Introduction

Many optical transmission experiments do not include forward error correction (FEC). Instead, *thresholds* are used to decide whether transmission was successful or not. The most commonly used threshold is the pre-FEC bit error rate (BER). This threshold gives accurate predictions if some interleaving assumptions are fulfilled, if the FEC is binary, and if the decoder is based on hard decisions. However, pre-FEC BER fails at predicting the post-FEC BER of binary *soft-decision* FEC, as recently shown in [1]. In this paper, we investigate the performance prediction of *nonbinary* (NB) soft-decision FEC (NB-FEC) and argue that the correct threshold in this case is the mutual information (MI).[1]

The generalized mutual information (GMI) was proposed in [1] as a metric to characterize the performance of binary soft-decision FEC. The rationale for using the GMI was that it is an *achievable rate* for bit-interleaved coded modulation (BICM), often employed as a pragmatic approach to coded modulation (CM). For square quadrature amplitude modulation (QAM) constellations, BICM operates close to capacity with moderate effort. However, for most nonsquare QAM constellations, BICM results in unavoidable performance penalties. For these modulation formats, other CM schemes such as NB-FEC [3] and multi-level coding with multi-stage decoding [4] can be advantageous.

In this paper, the MI is shown to be the correct threshold for a CM scheme based on NB low-density parity-check (LDPC) codes. This is verified in both an additive white Gaussian noise (AWGN) simulation and in an optical experiment using 8-QAM constellations. Other thresholds (including the GMI) are shown to fail in this scenario.

2. Thresholds for Nonbinary FEC

Figure 1 shows the NB-CM scheme under consideration. The data bits are mapped to NB symbols from GF(2^m), encoded by an NB-FEC with rate R, and then mapped to constellation symbols from the set $\mathscr{X}$, where $|\mathscr{X}| = 2^m = M$. The constellation symbols are transmitted with equal probability $1/M$ through an "optical channel" (which includes DACs, filtering, ADCs, DSP, interleaving, etc.). For each symbol y, the soft symbol demodulator computes M likelihoods $p(y|x_i)$, where $x_i \in \mathscr{X}$, $i = 1,2,\ldots,M$ (or $M-1$ nonbinary LLRs), which are passed to an NB-FEC decoder.

Neglecting channel memory and assuming independent symbols, the MI is given by

$$I(X;Y) := \frac{1}{M}\sum_{x\in\mathscr{X}}\int_{y\in\mathbb{C}} p(y|x)\log_2\left(\frac{M\cdot p(y|x)}{\sum_{\tilde{x}\in\mathscr{X}} p(y|\tilde{x})}\right), \tag{1}$$

where $\mathbb{C}$ is the complex I-Q plane and $p(y|x)$ is an average conditional channel probability density function (PDF) [5].

The MI in (1) is a lower bound on the largest achievable rate for a given modulation, with equality if the channel is memoryless and ergodic [5]. If sufficiently long symbol-wise interleaving is applied (within the equivalent "optical channel"), we can assume that these conditions hold. The MI can be efficiently estimated using the *auxiliary channel lower bound* $\underline{I}(X;Y) := \mathbb{E}\left\{\log_2\left(\frac{M\cdot q(Y|X)}{\sum_{\tilde{x}\in\mathscr{X}} q(Y|X=\tilde{x})}\right)\right\} \leq I(X;Y)$ [6, Sec. 2], where $\mathbb{E}\{\cdot\}$ denotes expectation over $p(y|x)$ and $q(y|x)$ is a conditional channel PDF approximating $p(y|x)$. If $q(y|x) = p(y|x)$, we have $\underline{I}(X;Y) = I(X;Y)$. As the noise in uncompensated coherent optical fiber communication tends to be Gaussian [7], a good choice is $q(y|x) = \frac{1}{\pi\sigma^2}\exp(-|y-x|^2/\sigma^2)$. In this paper, we use a *Kernel Density Estimator* (KDE) [8] to get a good estimate of the conditional channel PDF $q(y|x) \approx p(y|x)$ from the measurements, which yields $\underline{I}(X;Y) \approx I(X;Y)$.

[1] The MI was in fact previously introduced in [2] to assess the performance of differentially encoded quaternary phase shift keying and it was found that it can be better performance indicator than the pre-FEC BER in that application.

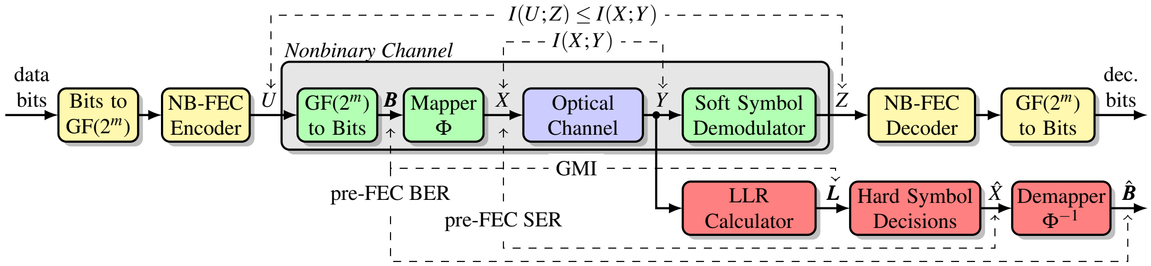

Fig. 1: System model of optical transmission based on NB-CM and the measurement of various system parameters.

In the next section, the accuracy of the MI as a decoding threshold will be compared against predictions based on the GMI [1], the pre-FEC BER $\frac{1}{m}\sum_{i=1}^{m} \Pr\left\{\hat{B}_i \neq B_i\right\}$, and the pre-FEC symbol error rate (SER) $\Pr\left\{\hat{X} \neq X\right\}$. These quantities are schematically shown at the bottom of Fig. 1. We immediately see that only the MI is directly connected to the NB-FEC decoder, and thus is the most natural choice. In particular, the transmitter in Fig. 1 uses a GF(2^m)-to-bit mapper followed by a bit-to-symbol mapper $\Phi(\boldsymbol{b}) = x$, which maps the vector of bits $\boldsymbol{b} = (b_1, b_2, \ldots, b_m)$ to a constellation symbol $x \in \mathscr{X}$. These blocks are included only so that the GMI and pre-FEC BER can be defined (and calculated) but have no operational significance for the NB-CM system under consideration, as U can be directly mapped to X. At the receiver side, additionally logarithmic likelihood ratios (LLRs) are calculated ($\boldsymbol{L}$), a hard-decision on the symbols is made ($\hat{X}$), which leads to a hard-decision on the bits ($\hat{\boldsymbol{B}}$).

In this paper, the GMI was calculated as described in [1, Sec. III-D]. The bit labeling used in the mapper Φ affects both the GMI and pre-FEC BER, however, it has no impact on the actual performance of the system (nor on the MI or pre-FEC SER). Note that the NB channel in Fig. 1 is characterized by the MI $I(U;Z)$. It is however often more natural in an optical transmission experiment to directly measure $I(X;Y)$ without the need to invoke the *soft symbol demodulator*. The goal of this paper is to show that we can use $I(X;Y)$ as a proxy to characterize the *NB channel* between U and Z and to predict the performance of the NB-LDPC decoder. Generally, we have $I(U;Z) \leq I(X;Y)$ with equality if the soft symbol demodulator computes likelihoods using the actual averaged channel transition PDF $p(y|x)$. In the soft symbol demodulator, we assume a Gaussian channel PDF to simplify practical hardware implementation, such that in general, the inequality holds. Note that directly measuring $I(U;Z)$ is generally not an easy task, as Z is a random variable defined over the M-dimensional probability space. However, we found that the $I(X;Y)$ generally gives a good approximation to $I(U;Z)$. Finding easy ways to compute $I(U;Z)$ when a mismatched PDF is assumed in the soft demodulator is part of our ongoing work.

3. Experimental Transmission of Various 8-QAM Formats

We consider the three 8-QAM constellations shown in Fig. 2, where the bit-mapping that maximizes the GMI is also shown [9]. Three quasi-cyclic NB-LDPC codes with rates $R \in \{0.7, 0.75, 0.8\}$ (FEC overheads of $\approx 43, 33, 25\%$) defined over GF(2^3) with regular variable node degree of 3 (check-concentrated) and girth 8 are considered. Decoding takes place using 15 iterations with a row-layered belief propagation decoder. These codes are conjectured to be *universal*, i.e., their performance is expected to be independent of the actual channel.

In Fig. 2 we show the post-FEC SER as a function of the four different performance metrics studied in this paper. Changing the constellation for a given code can be interpreted as changing the nonbinary channel in Fig. 1. The results in Fig. 2 were obtained for the AWGN channel and clearly show that only the MI can be used as a reliable threshold. In particular, for a post-FEC SER of 10^{-4} (horizontal lines in Fig. 2), the obtained MI thresholds are $T_{0.7} = 2.31$, $T_{0.75} = 2.43$, and $T_{0.8} = 2.55$, for $R = 0.7$, $R = 0.75$, and $R = 0.8$, respectively.

To validate the AWGN results in Fig. 2, we now consider a dual-polarization 41.6 Gbaud system. The three 8-QAM constellations of Fig. 2 were generated and tested using a high-speed DAC in a back-to-back configuration. A root-raised cosine pulse shaping (roll-off factor 0.1) signal was generated as described in [9] and two code rates ($R = 0.7$ and $R = 0.8$) were considered, giving net data rates of approximately 174 and 200 Gbit/s.

The MI as a function of the OSNR for the three constellations is shown in Fig. 3 (left), where the constellation $\mathscr{C}_3$ shows a clear superiority in terms of MI. In this figure, we also show the MI thresholds $T_{0.7} = 2.31$ and $T_{0.8} = 2.55$ from Fig. 2. These MI thresholds are then used to determine equivalent OSNR thresholds for all three modulation formats (see vertical lines in Fig. 3 (left)). The measured data was then used to perform NB-LDPC decoding. The obtained results are shown Fig. 3 (right), which show a good match between the MI thresholds obtained for the AWGN channel and the actual performance of the codes in the experiment. Note that, as $I(X;Y) \geq I(U;Z)$, the OSNR thresholds based on the experimentally measured $I(X;Y)$ actually lower bound the thresholds computed from $I(U;Z)$, which explains the slight discrepancies in Fig. 3.

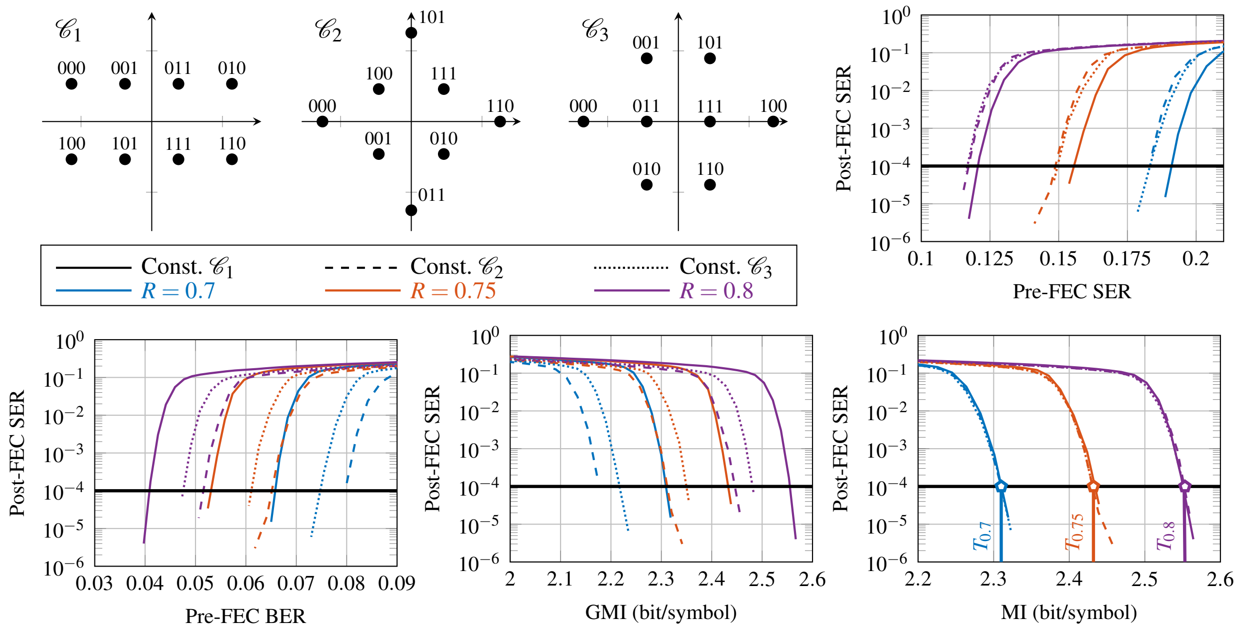

Fig. 2: Three different 8-QAM constellations used in the experiments taken from [9]. The numbers adjacent to the constellation points give the GMI-maximizing bit labeling. The plots show the use of the four different performance metrics for NB-LDPC codes

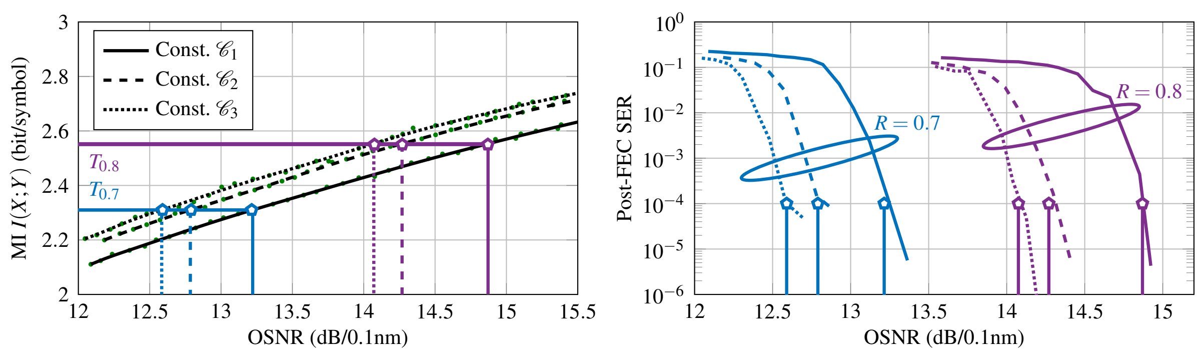

Fig. 3: Empirically (green markers) and interpolated (lines) MI curves (left) and results after actual decoding with an NB-LDPC decoder (right).

4. Conclusions

Different performance metrics for coded modulation based on capacity-approaching *nonbinary* codes were compared. It was shown in simulations and experiments that an accurate predictor of the performance of these codes is the mutual information. Uncoded metrics such as pre-FEC BER and pre-FEC SER were shown to fail. The GMI also failed for nonbinary codes, but still remains a good performance indicator for BICM with *binary* soft-decision FEC.

Acknowledgments

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