Spectral efficiency estimation in periodic nonlinear Fourier transform based communication systems

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Abstract: We evaluate, for the first time, the achievable spectral efficiency of periodic nonlinear Fourier transform based communication systems with hard decision FEC and modulated perturbed plane waves with high order QAM formats, e.g 32QAM-512QAM.

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1. Introduction

Nonlinear Fourier transform (NFT) has been considered extensively in recent years as a novel technique to combat the fiber nonlinear impairments [1, 2]. The main motivation behind the NFT transmission ideology is the fact that nonlinear signal spectrum propagates linearly along the fiber links, which leads to the possibility of developing simple nonlinear equalizers.

Even though the mathematical foundation of NFT is well developed for two basic variants, namely for signals with returning-to-zero and periodic boundary conditions, most of the NFT-based communication systems up to date have been designed with returning-to-zero signals. These include multi-soliton and nonlinear inverse synthesis transmissions with the modulations of discrete and continuous nonlinear spectra, respectively [2–4]. In addition, the first fully modulated NFT-based system using the full nonlinear spectra for data transmission has also been demonstrated in [5].

However, serious and yet unsolved problems of using imposing return-to-zero boundary condition are the substantial constraint on continuous transmission and a large computational complexity associated with the large processing window. On the other hand, similarly to conventional linear communications, all these aforementioned problems can be eliminated by using the periodic signal which offers the opportunity of controlling the time duration and bandwidth of the signal. Recently, the first periodic nonlinear Fourier transform (PNFT)-based system has been proposed in [6] using one dimensional constellation which allowed for high order PAM formats to be transmitted. Soon after that, a simple method for constructing two dimensional signals with high order QAM constellations without using the highly complicated inverse PNFT was briefly demonstrated in [7]. In this work, we extend the modulation concept of [7] and consider transmission performance of PNFT-based systems with modulated perturbed plane waves with high order QAM formats. For the first time, we estimate the achievable spectral efficiency (SE) of such communication systems and showing that a SE above 2.5 bits/s/Hz can be achieved over a distance of 1000 km.

2. Basic of PNFT and PNFT-based systems

We use the normalized nonlinear Schrödinger equation (NLSE) that is the master model for optical fibre channel, $i\,q_z + q_{tt} + 2q|q|^2 = n$, where $n(t,z)$ is the effective circularly symmetric complex Gaussian noise introduced by amplifiers with $\langle n(t,z), n(t',z')\rangle = D\delta(t-t', z-z')$, D characterises the noise power, $\delta(\cdot)$ is the Dirac delta-function and $q(t,z)$ is the slowly varying envelope of the electromagnetic field (see details, e.g. in [1, 4]). The nonlinear spectrum (NS) is found through spectral analysis of Zakharov-Shabat problem (ZSS):

$$\begin{bmatrix} i\partial_t & q(t,z) \\ -q^*(t,z) & -i\partial_t \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \lambda \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}. \tag{1}$$

In the periodic case [7], those points of the spectrum (eigenvalues of ZSS (1)) in which the associated eigenfunction, $\Phi = [\phi_1, \phi_2]^T$, is periodic or anti-periodic, are called *main spectrum* (MS). MS is the invariant part of the NS which along with the changing (in z) part of NS, *auxiliary spectrum*, provides a full NFT representation for signal. To simplify the calculations and design, here we omit the auxiliary spectrum. The procedure of calculating MS from the

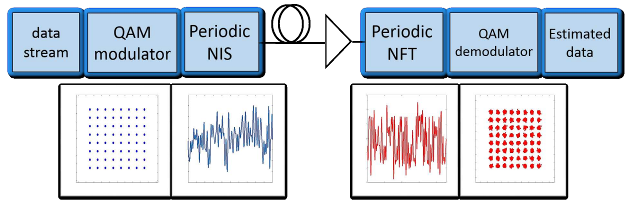

Fig. 1: NFT-based communication system, data is mapped on MS at the transmitter and extracted through direct transform at the receiver.

signal (*direct transformation*) can be carried out using the Ablowitz-Ladik routine [8]. Similarly, the signal can also be contracted from its NS through the *inverse transformation*. The idea of using PNFT in communication systems is to construct signals with MS modulated by arbitrary data. Since MS remains unaltered during propagation, one can easily retrieve the mapped data by finding MS of the received signal (see Fig. 1) . Unfortunately, unlike the direct transformation, which is quite well established with fast and efficient numerical methods, there is still a lack of an appropriate method to perform the inverse transformation stage. As a result, it is of great interest to elaborate an alternative to the inverse transformation procedure, especially to accommodate a two-dimensional constellation for the transmitted signal.

Let us start from any MS in the form of the middle panel in Figure 2, consisting of a purely imaginary nondegenerate point plus some complex points, λ_i, symmetric around a few imaginary points, λ_n^0. When λ_n^0s are degenerate points of a plane wave this could be assumed as the NS of a plane wave modulated by some (as many as λ_n^0s) sinusoidal waves with amplitude ε_n. In the finite-gap integration theory any solution of the NLSE can be read as the ratio of two Riemann theta functions. Riemann theta function is the multidimensional generalization of the ordinary FT (for definition of the parameters and more information see [7]). So we can construct the perturbed plane wave using the available formula in [7] which produces a signal (Fig. 2) whose MS is the one in the middle panel of Fig. 2 to the second order of $|\varepsilon|$. Hence the smaller the aperture between new nondegenerate points, the better the accuracy. Since ε is a complex number we can draw it from an arbitrary constellation like the one in the right panel of Fig. 2.

3. Simulation results and discussion

We consider in this work a fiber link with standard single mode fiber ($\alpha = 0.2$dB/km, $\beta_2 = -21.67\,\text{ps}^2$/Km and $\gamma = 1.27\,\text{W}^{-1}\text{Km}^{-1}$) with ideal Raman amplification. The propagation of signal over fiber link was simulated using the well-known split-step Fourier method, where the ASE noise was added distributively (every 50 km) along the link. To investigate the transmission performance of the considered PNFT-based systems, we construct signals with around 0 dBm power and 2 GHz of bandwidth with different modulation formats and evaluate the BER at the end of the links by directly counting the mismatches between the transmitted and received bit streams. For the investigated systems, we observed several sources of deterministic errors due to the approximation and of the Riemann theta function's parameters as well as the numerical methods associated to the Riemann theta function calculation. However, due to the deterministic nature of such numerical distortions (noticeable in the receiver constellation shown in the left panel of Fig. 3), their impacts can be effectively pre- or post-compensated at the transmitter and receiver using pilot symbols. In this work, these compensation steps were performed at the receiver with the help of pilot symbols providing less than 3% overhead.

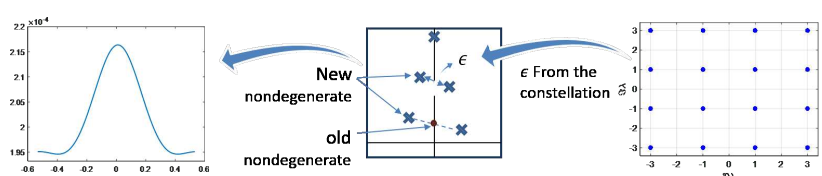

Fig. 2: A drawn complex point from an arbitrary constellation in the right can determine ε, the aperture between new nondegenerate points (in the middle panel), which makes the MS of the signal in the left.

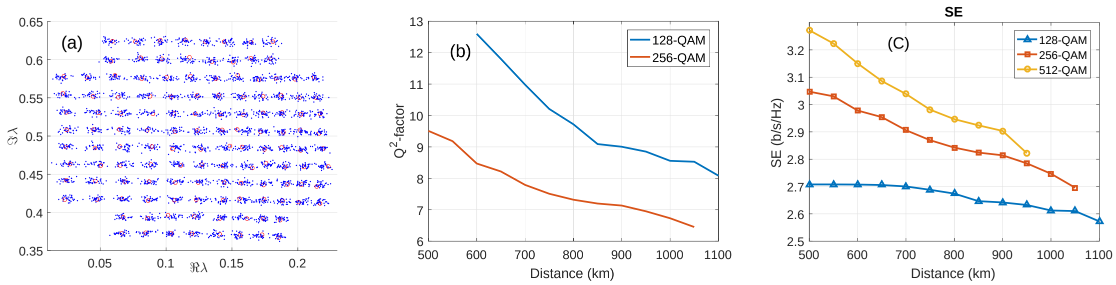

Fig. 3: a) Receiver constellation for 128-QAM at z=1000 km, b) Q^2-factor with signal power of -0.5 dBm, and c) SE of 128/256/512-QAM.

The Q-factors as functions of the distance for 128QAM and 256QAM formats are depicted in the middle panel of Fig. 3, showing clearly that communications with very high order QAM formats can be effectively achieved in PNFT-based systems. However, PNFT-based system requires cyclic prefix (CP) extension in a similar way to the conventional OFDM transmissions for preventing inter-symbol interference. The use of CP reduces the net data rate and the resulted SE of the system. In this work, we estimated the SE of investigated PNFT-based system by firstly passing the received signals through a low-pass filter with 2 GHz of bandwidth. After that by assuming the use of hard decision forward error correction we calculated the achievable rate in bits/symbols. Next, we converted the achievable rate into the net data rate taking into account the CP and finally the SE was estimated within 2 GHz of bandwidth. The obtained SE versus distance result is depicted in the right panel of Fig. 3, showing the tendency of higher SE for higher QAM formats. In particular, at a distance of 500 km, the SEs with 128QAM, 256QAM and 512QAM are $\sim$ 2.7, 3 and 3.27 bits/s/Hz. At $z = 1000$ a SE of 2.75 bits/s/Hz can be achieved with 256QAM format. We also notice that relation between the SE and reach is not trivial since for different distances the cyclic extension overheads are different.

4. Conclusion

We have demonstrated that narrowband PNFT-based transmission can effectively support high order QAM formats, such as 128QAM, 256QAM and 512QAM. By taking into account the CP for preventing inter-symbol interference and a limited bandwidth of 2 GHz, we have calculated, for the first time, the achievable SE of PNFT-based systems. The obtained result clearly indicates that a SE of 2.75 bits/s/Hz can be achieved over 1000 km of transmission distance.

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