Nonlinear signal transformations: path to capacity above the linear AWGN Shannon limit

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***Abstract*—We present a methodology for simultaneous optimization of modulation format and regenerative transformations in nonlinear communication channels. We derived analytically the maximum regenerative Shannon capacity, towards which any regenerative channel tends at high SNR and large number of regenerators.**

I. INTRODUCTION

The nonlinear communication channels are conceptually challenging and much less studied than the linear channels. All-optical regeneration is an example of such nonlinear channel. Recently schemes capable to regenerate multilevel advanced modulation formats have been demonstrated [1-6]. A number of experimental demonstrations were reported showing the cascadability of various regenerative schemes [6-9]. However, the impact of regeneration on channel capacity remains largely unstudied [10].

Here we present an optimization procedure for nonlinear regenerative transformations – regenerative mapping, which reflects the interplay between signal modulation and nonlinearity. The proposed scheme is general and can be applied to any regenerative model. As a result, the noise is suppressed, which leads to higher Shannon capacity above the seminal linear Shannon limit. Furthermore, the analytical expression for the maximum regenerative capacity – regenerative Shannon limit - is derived.

II. REGENERATIVE MAPPING

Here by a term "regenerator" we refer to a device that performs a nonlinear signal transformation characterized by a smooth nonlinear transfer function (TF) (see Fig. 1a), which creates attractive regions around the alphabet – a set of points used for signal modulation. Therefore, the alphabet points are required [11-12] to be stationary $T(x^*) = x^*$ and stable $|T'(x^*)| \leq 1$. Finally, the optimum positioning of the alphabet point is in the center of the attraction region $T''(x^*) = 0$.

III. REGENERATIVE TRANSFORMATIONS

In Fig. 1a) it is demonstrated on the example of the regenerative Fourier transform (RFT or sine-mapping): $y = T(x) = x + \alpha \sin(\beta x)$ that by varying transformation parameters one changes the positioning of the alphabet points (shown by vertical lines) and the slope of the function (which defines the strength of noise suppression). RFT represents the

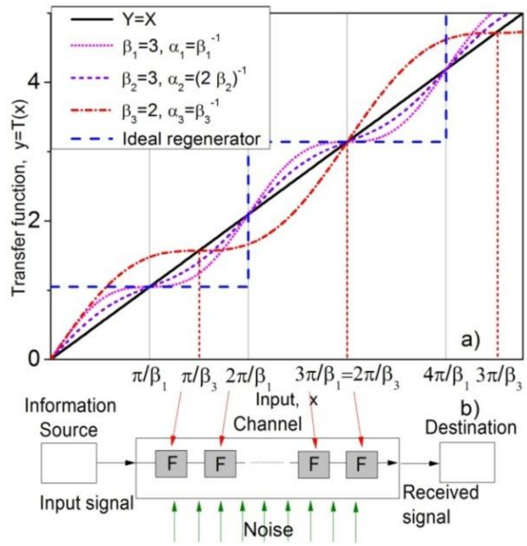

Fig.1. a) Transfer function of an ideal regenerator (piece-wise dashed line) compared to smooth regenerative Fourier transform plotted for different parameters. b) Basic scheme of a regenerative channel: regenerative filters (denoted by F) are placed equidistantly along the transmission line.

first terms of the Fourier expansion of the ideal regenerator; therefore, it is the closest smooth approximation of the ideal regenerator and enables multilevel signal regeneration.

Also, there are a variety of different all-optical regenerative schemes. In particular, it was experimentally demonstrated that nonlinear optical loop mirror enables multilevel amplitude regeneration [3-5], whereas phase sensitive amplification allows multilevel phase regeneration [1-2]. Combined together they offer simultaneous suppression of phase and amplitude distortions [5]. Moreover, a number of experimental results for cascadability of these schemes were reported [6-9].

To increase channel capacity regenerators need to handle multilevel modulation formats, however, since nonlinearity plays crucial role in regenerative transmission, one has to optimize simultaneously the regenerative element and modulation using the regenerative mapping technique. Also, circular QAM constellations [13] are beneficial for regenerative transmission with phase and/or amplitude regeneration [5,14]. Thus, a regenerative element is essentially a filter, which squeezes signal perturbation by noise or destructive nonlinearity. This is different from the Decode-and-Forward model, where hard decision is performed by in-line elements, which has a piece-wise TF (further referred as ideal regenerative TF). Therefore, a regenerative channel is a new channel type, where nonlinearity is used for improving signal transmission and is fundamentally different from Decode-and-Forward model as no hard-decision is required in-line. Instead, by placing more regenerative filters in cascades along the transmission line, one increases the strength of noise squeezing and, consequently, the channel capacity will be higher.

On the other hand, the Shannon capacity is limited by error propagation – an error, which occurs when the signal distortion is larger than the decision region of a regenerative filter. This effect sets an upper bound on the regenerative Shannon capacity – regenerative Shannon limit. Moreover, at high SNR and/or large number of filters in cascades a regenerative channel with arbitrary smooth regenerative TF will tend to the regenerative Shannon limit. Thus, the regenerative Shannon limit shows the maximum gain in Shannon capacity that one can expect by employing regeneration. Here we calculate the main order of Shannon capacity by omitting effects of dispersion and destructive nonlinearity, which can be incorporated into analysis by using the framework of perturbation theory [15]. The cascadability of different regenerative schemes was investigated in [6-9].

IV. REGENERATIVE SHANNON LIMIT

The capacity analysis of the system with the ideal regenerators defines the upper bound of regeneration efficiency. The ideal regenerators assign each transmitted symbol to the closest element of the given alphabet (see Fig. 1a). Consider a regenerative channel with R identical nonlinear filters placed along the transmission line. The signal transmission (for simplicity, signal propagation between the regenerators is assumed to be linear) is distorted by an additive white Gaussian noise that is uniformly distributed along the line.

The analytical formula for the regenerative limit were derived [11], in particular at high SNR one can observe the constant gap between the n-dimensional linear Shannon limit and the regenerative capacity:

$$\Delta C_R = \frac{n}{2}\log_2\left(\frac{2\pi e N}{d_{opt}^2}\right) + \frac{n}{2}\frac{Re^{-\Delta^2}}{\Delta\sqrt{\pi}}\log_2\left(\frac{Re^{-\Delta^2}}{\Delta\sqrt{\pi}}\right)$$

where AWGN noise power is denoted by N, dimensionless parameter by $\Delta = d_{opt}\sqrt{R/8N}$, and the optimum decision area size - $d_{opt} = 8N\kappa/R\mathrm{W}(e^2R^2/8\pi\kappa)$, here $\kappa = 1 + 10/R$ and W is Lambert W function.

Figure 2 depicts the analytically calculated formula (shown by the black dotted lines, which is in excellent agreement with the numerical simulations (shown by colored solid lines). The maximum regenerative capacity gain (that is, the maximum regeneration efficiency) is observed for the binary channel. This reflects the trade-off between the system complexity and capacity improvement. With the increasing number of regenerators the peak of the capacity gain is observed at smaller SNR. Therefore, employing a low-SNR regime and using regeneration, one can achieve higher transmission performance with lower energy consumption.

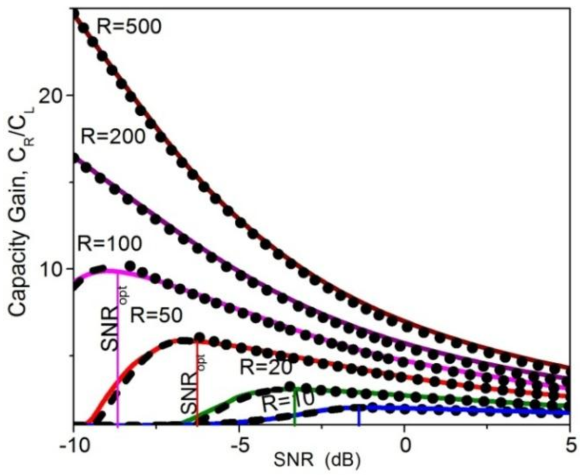

Fig.2. The maximum gain in Shannon capacity, which can be achieved placing a different number of regenerators (R) in-line.

V. CONCLUSIONS

We have calculated the maximum gain of Shannon capacity due to regeneration. We have proved that nonlinear signal transformations optimized according to the proposed regenerative mapping technique enable to achieve nonlinear Shannon capacity higher than the Shannon limit for linear additive white Gaussian noise channel. This explores a fascinating new feature of the interplay between stochastic processes and system nonlinearity.

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