

# Regenerative Fourier transformation for dual-quadrature regeneration of multilevel rectangular QAM

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We propose a new non-linear optical loop mirror based configuration capable of regenerating regular rectangular QAM signals. The scheme achieves suppression of noise distortion on both signal quadratures through the realization of two orthogonal regenerative Fourier transformations. Numerical simulations show the performance of the scheme for high constellation complexities (including 256-QAM formats).

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The increasing capacity demand in optical networks has required the use of complex quadrature amplitude modulated (QAM) formats characterized by multilevel encoding in amplitude and phase. Despite its spectral efficiency benefits the QAM modulation is highly sensitive to fibre link distortions [1] and thus unable to support transmission over ultra long haul distances. All-optical regeneration, by enabling the periodic restoration of the signal quality along the link, has the potential to address this issue and to match the diverse requirements of high capacity and long haul networking.

Despite the recent developments in the design and implementation of new all-optical regeneration techniques, there is still a lack of a generic approach to deal with QAM constellations of any order offering simultaneous suppression of amplitude and phase noise. The phase quantizer proposed in [2], is limited to phase shift keyed (M-PSK) encoded waveforms and it does not suppresses the amplitude distortion. Combining the scheme with a phase preserving non-linear optical loop mirror (NOLM) [3] amplitude noise suppression could be also provided [4], however, dual-quadrature regeneration for more than two amplitude levels has not been shown yet [5]. A more advanced NOLM scheme has been proposed in [6] to deal with an increased number of amplitude levels, but it is still limited to circular-QAM signals. As rectangular QAMs are becoming the prevailing format in optical communications suitable regenerator solutions will be accordingly required.

Towards this direction, removal of the distortion from both signal quadratures has been shown in [7], by combining self-phase modulation induced phase shifting and

PSA based quadrature squeezing to regenerate each amplitude level. The scheme is highly efficient but its complexity scales proportionally to the number of amplitude levels at each signal quadrature. Recently in [8], we introduced the regenerative Fourier Transform (RFT) and showed that the 1<sup>st</sup>-order Fourier expansion of an ideal step-wise regenerator enabling an infinite number of regenerative levels on each signal quadrature. When applied in transmission lines the RFT can provide Shannon capacities beyond the linear Shannon limits [8], and as such, its all-optical realizations are highly desirable for future networks.

In this paper, we propose a new all-optical design of the RFT, based on the use of a novel NOLM configuration. Contrary to the traditional phase insensitive schemes [4, 6, 11], here, the non-linear phase shift is achieved through coherent mixing of the signal with a local pump. This creates a sinusoidal transformation on the amplitude and not on the power of the input field. Furthermore, unlike [9, 10], the interferometer operates in an asymmetric mode, i.e. the pump interacts only with the counter-propagating wave, allowing for the required beating of the original signal with its non-linear transformed to realize the RFT. This unique combination of properties makes the proposed scheme compatible with conventional rectangular QAM signals of any duty cycle and constellation order with performance close to that of ideal regenerator.

The RFT transfer function is the first order expansion in Fourier series of the step-wise transfer function (TF), therefore, it represents a smooth and close approximation of an ideal regenerator. Indeed, a step-wise TF with step  $2x_0$  is given by:  $[x/x_0]$  where  $[ ]$  is the integer part of the variable. One can write:

$$y_{\text{ideal}} = \left[ \frac{x}{x_0} \right] = x - \text{mod}(x, 2x_0) + x_0 \quad (1)$$

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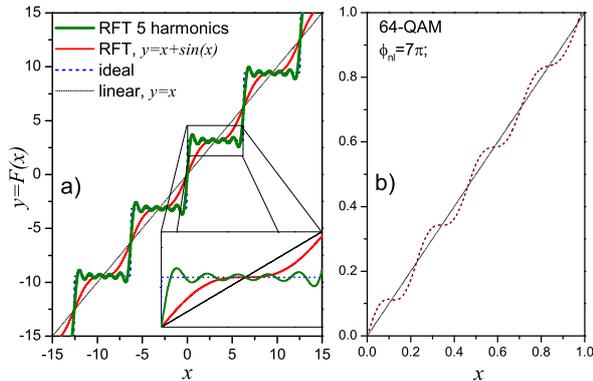


Fig. 1. Transfer functions of RFT compared with step-wise response of an ideal regenerator a) as given by Eq. 2 for different number of harmonics and b) its optical realization by the scheme of Fig. 2 for regenerating 64-QAM signals.

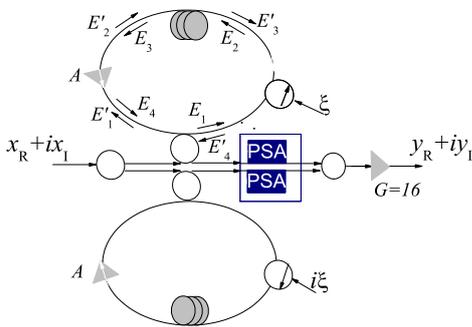


Fig. 2. Scheme: The signal is split in two copies and each one of them is launched to an asymmetric NOLM configuration. At the interferometer, the counterclockwise-propagating signal is mixed with a strong pump pulse and undergoes a non-linear transformation, while the clockwise-propagating signal remains almost unaffected. Afterwards, both waves are coupled together and their real part is selected by a PSA element. The regenerated signal quadratures are finally coupled together at the output of the MZI.

By expanding the mod operator in Fourier series we get:

$$y_{\text{ideal}} = \begin{bmatrix} x \\ x_0 \end{bmatrix} = x - \left( x_0 - \frac{2x_0}{\pi} \sum_{k=1}^{\infty} \frac{\sin\left(\frac{\pi k x}{x_0}\right)}{k} \right) + x_0 \quad (2)$$

The RFT allows to approach an ideal step-wise TF very closely when increasing the number of harmonics (see Fig. 1a), whereas after keeping the first-order term and notating  $\pi/x_0$  as  $\beta$  and  $2x_0/\pi$  as  $\alpha$  one obtains:

$$y_{\text{RFT}} = F(x) = x + \alpha \sin(\beta x) \quad (3)$$

The transfer function is symmetric with respect to zero  $F(x) = F(-x)$  and is applied independently to both quadratures, i.e.  $y_{R,I} = F(x_{R,I})$ , where  $x_R$  is the real

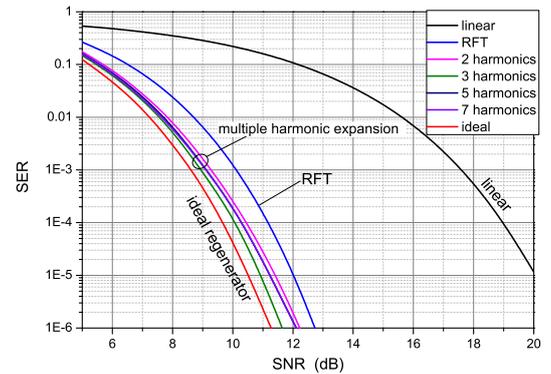


Fig. 3. SER as a function of the SNR for 16-QAM signals for 10 equidistant regenerators with various TF.

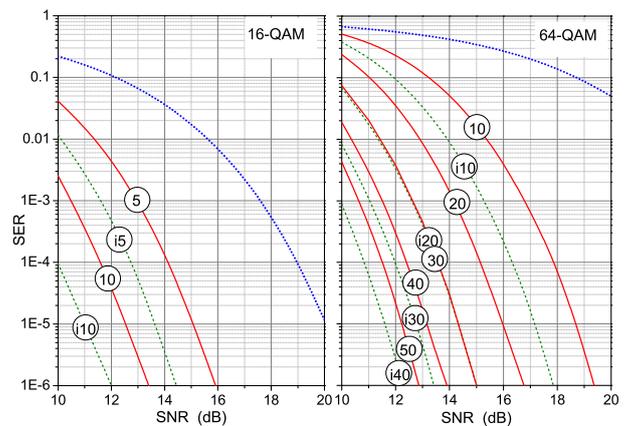


Fig. 4. In red solid lines the SER as a function of the SNR, for 16- and 64-QAM signals, and for different number (in circle) of cascaded 1<sup>st</sup> order RFT regenerators. In green dashed lines the SER when a number (denoted also by i) of ideal regenerators is considered. Finally, in blue dotted lines the SER of corresponding linear channels, is shown for reference

and  $x_I$  the imaginary part of the optical signal. In addition, the  $\alpha, \beta$  parameters define the location and the efficiency of the regenerative areas on the applied transformation. Multiple equidistant plateaus exist at the points  $x_{R,I}(k) = (2k+1)\pi/\beta$   $k \in \mathbb{Z}$  for the condition  $\alpha\beta = 1$ , see Fig. 1a), however, noise suppression can be achieved also for suboptimal parameters in the range  $0 < \alpha\beta < 2$  [8].

To realize this transformation the scheme of Fig.2 is proposed. It consists of a symmetric Mach-Zehnder Interferometer (MZI) with a NOLM and a phase sensitive amplifier (PSA) [13] in each arm to regenerate independently the two signal quadratures. The scheme can be reduced to a single NOLM+PSA configuration by exploiting the polarization dimension.

In each NOLM, the incoming signal is split in counter- and clockwise parts (here and further all the couplers

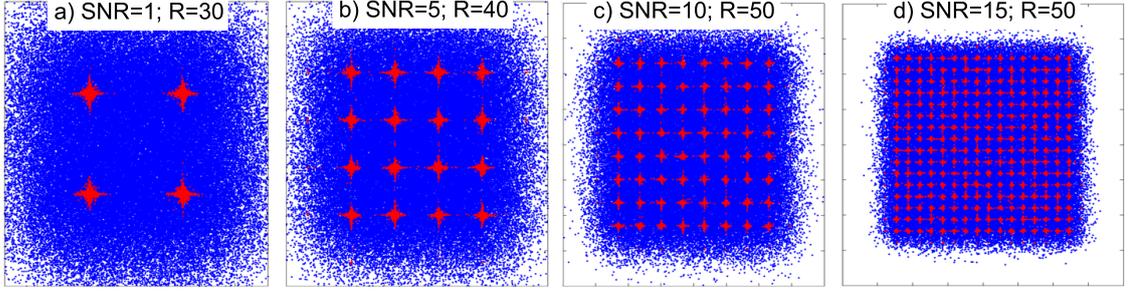


Fig. 5. Constellation diagrams (normalized to average signal power) of rectangular (a) 4-, (b) 16-, (c) 64-, and (d) 256-QAMs signals after propagation through a transmission line with (red) or without (blue)  $R$  equidistantly placed 1<sup>st</sup> order RFT regenerators. The depicted SNR (dB) value characterizes the signal-to-noise ratio of the corresponding linear system.

have 1 : 1 coupling ratio), which are given by:

$$E_1 = \frac{i}{2}(x_R + ix_I), \quad E'_1 = \frac{1}{2}(x_R + ix_I) \quad (4)$$

The counter-clock propagating signal is coherently coupled with a pulsed pump wave of variable duty cycle and peak amplitude  $\xi$ , where we assume  $\xi \gg x_{R,I}$ :

$$E_2 = \frac{1}{\sqrt{2}} \left[ \frac{i^2}{2}(x_R + ix_I) + \xi \right], \quad E'_2 = E'_1 \quad (5)$$

and subsequently transmitted via length  $L$  of highly non-linear fiber (HNLF) with zero dispersion, resulting in:

$$E_3 = E_2 e^{i\gamma L[|E_2|^2 + 2|E'_2|^2]} \quad (6)$$

we choose the pump to be much stronger than the signal  $\xi \gg x_{R,I}$  and we consider the signal-signal interactions to be small (which is usually the case)  $\gamma L x_{R,I}^2 \ll 1$ , also we compensate the constant non-linear phase shift  $\gamma L \xi^2/4$  (alternatively, one can choose parameters so that  $\gamma L \xi^2/4 = 2\pi n$ ,  $n \in \mathbb{Z}$ ), so that

$$E_3 \simeq \frac{\xi}{\sqrt{2}} e^{-i\xi x_R \gamma L/2} \quad (7)$$

Similar for the clockwise-propagating wave we omit  $\gamma L x^2 \ll 1$  terms and compensate the constant non-linear phase shift  $\gamma L \xi^2/4$ , since  $\langle x_{R,I} \rangle = 0$ , we derived

$$E'_3 = E'_2 e^{i\gamma L[|E'_2|^2 + 2|E_2|^2]} \simeq E'_2 \quad (8)$$

Next the counter-clockwise propagating wave is attenuated by a directional attenuator ([14]) with attenuation coefficient  $A$ , meanwhile, the counter-propagating wave remains unchanged  $E'_4 = E'_3$ . Coupling  $E_4$  and  $E'_4$  and selecting, with the use of the subsequent PSA, their real parts results in

$$E_R = \text{Re} \left( \frac{E'_4 + iE_4}{\sqrt{2}} \right) = \frac{1}{\sqrt{8}} \left( x_R + \sqrt{A} \xi \sin \left( \frac{\xi x_R \gamma L}{2} \right) \right) \quad (9)$$

Similar derivations will hold for the second NOLM element with clockwise and counter clockwise waves interchanged resulting in the nonlinear transformation of the imaginary signal part.

The transformed quadratures are then coupled together and amplified by gain  $G = 16$  to restore the original signal power. This results in the 1<sup>st</sup> order RFT applied independently to both quadratures:

$$y_R + iy_I = \frac{\sqrt{G}}{\sqrt{2}} (E_R + E_I) = F(x_R) + iF(x_I) \quad (10)$$

$$F(x_{R,I}) = x_{R,I} + \alpha \sin(\beta x_{R,I}), \quad \alpha = \xi \sqrt{A}, \quad \beta = \frac{\xi \gamma L}{2}$$

For regenerating 4-, 16, 64, and 256-QAM signals non-linear phase shifts of  $\pi$ ,  $3\pi$ ,  $7\pi$ ,  $11\pi$  are respectively required, as an example TF for regenerating 64-QAM signals is shown in Fig. 1b. This can be achieved by various combinations of fiber length  $L$  and pump power levels. By varying the HNLF length (which affects the value of  $\beta$ ) one changes the positioning and number of regenerative levels, whereas by optimizing the attenuation coefficient  $A$  (this defines  $\alpha$ ) it allows to adjust steepness of the TF, and for  $\alpha\beta = 1$  one can obtain plateau levels. The output of the proposed regenerator deviates from the desired 1<sup>st</sup> order RFT for large values of amplitude, when the condition  $\xi \gg x$  is not satisfied.

In real fibre based implementations, stimulated Brillouin scattering (SBS) is expected to be a serious limiting factor if we assume operation with a continuous pump wave. This will necessitate to adopt and further improve recently proposed SBS suppression methods in the design and fabrication of the non-linear fiber medium [16]. Alternatively, SBS effect can be mitigated by operating with short pump pulses, duration of which should be a fraction of the symbol period and much shorter than the phonon lifetime ( $\sim 10$  ns). In that case the proposed RFT scheme would be a part of a 3R regenerator including additional subsystems for the timing synchronization of the pump pulses to the incoming signal pulses.

Subsequently, we evaluated the proposed RFT by simulating numerically the transmission of rectangular 4-, 16-, 64-, and 256 - QAM signals through a noisy channel of cascaded regenerators and compared their performance against the un-regenerated case. The signal distortion was modeled as additive white Gaussian noise distributed uniformly along the transmission line, whereas equidistant placement of the regenerators was considered. Our focus was on the noise suppression capabilities of the RFT scheme, therefore, the sections between the regenerators were represented by linear transmission lines ignoring any other nonlinear or dispersive effect. Timing jitter phenomena had been also neglected, i.e. by assuming a 3R functionality, which allowed us to represent the overall system as a cascade of noise elements and transfer functions and perform the simulations at one sample per symbol. Thus for simplicity, the point-to-point transmission system may be characterized by a single parameter, i.e. the signal-to-noise ratio (SNR), defined as the ratio of the average signal power at the input of the transmission system to the total power of the linearly added noise along the line [15]. This is equivalent to the signal to noise ratio of the corresponding linear system, i.e. at the absence of regenerative elements, and provides a common reference for a fair comparison of systems with different regenerative properties.

We calculated a symbol error rate (SER) by performing direct error counting on simulated  $2^{25}$  equiprobable symbols as a function of the SNR. As shown in Fig. 3 on the example of rectangular 16-QAM with 10 equidistantly placed regenerators with different TFs: increasing the number of RFT harmonics leads to better performance, however even single harmonic enables significant noise suppression. Further, varying the number of 1<sup>st</sup> order RFT regenerators we evaluated SER for rectangular 16- and 64-QAM signals. The results are depicted in Fig. 4 showing a dramatic improvement of the SER by several orders of magnitude. Note that the achieved regeneration gain scales with the constellation complexity of the signal. Comparing the SER performance when the 1<sup>st</sup> order RFT is used (see Fig. 4, red curves) to that of using an ideal step-wise regenerative transform we see that the SNR penalty is less than 2 dB. This means that the RFT can enable high regeneration efficiency and improve signal transmission without the need of hard decision based on the use of opto-electronic transponders (Tx/Rx) along the line.

Finally, in Fig. 5 we plotted the constellations of the different QAM signals after transmission via the linear channel (depicted in blue), and compared them to the case where a number of  $R$  regenerative elements have been included in the line (depicted in red), with  $R$  being equal to 30, 40 and 50. Moving to higher constellation orders an increased number of cascaded RFT elements and/or SNR had to be considered, since efficient regeneration could be achieved when the  $SNR/R$  ratio was less than the size of decision area (i.e. small-

est distance between the neighbouring alphabet points) [8]. The suppression of the accumulated noise on both signal quadratures was evident. Furthermore, the RFT induced squeezing affected the statistics of the residual noise, i.e. the circular symmetry of the initial Gaussian distribution (blue points) broke into a new axial symmetry (red points). Fig. 5 demonstrated also the scalability of the proposed scheme. By tuning only the pump power the transfer function of the regenerator could be adjusted to handle the different sizes of constellation.

We proposed a new all-optical regeneration approach for conventional multilevel rectangular QAM formats based on the use of two asymmetric NOLMs +PSA elements in MZI configuration. The scheme realizes an RFT transformation independently on the two signal quadratures enabling efficient suppression of noise distortion. Numerical simulations illustrated a successful cascading performance for the case of 4-, 16-, 64-, and 256-QAM signals.

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