Periodic nonlinear Fourier transform based optical communication systems in a band-limited regime

Morteza Kamalian, Jaroslaw E. Prilepsky, Son Thai Le, and Sergei K. Turitsyn

Aston Institute of Photonic Technologies, Aston University, Birmingham, B4 7ET, UK

kamaliam@aston.ac.uk

Abstract: A communication system based on the modulation of invariant quantities (eigenvalues) using periodic nonlinear Fourier transform (PNFT) is proposed. For the signal having 6 GHz bandwidth we evaluate the resulting BER and spectral efficiency.

OCIS codes: (060.1660) Coherent communications; (060.2330) fiber optics communications; (070.4340) Nonlinear optical signal processing

1. Introduction

Increasing demand for data rate in fiber-optic communication systems compels researchers to seek for new means of signal processing and communication system designs. One of the new directions in this field involves the use of the so-called nonlinear Fourier transform (NFT) for the fiber nonlinearity mitigation [1, 2]. The NFT is the method of analytical solution for a special class of nonlinear evolutionary equations, and it is particularly applicable for the lossless nonlinear Schrödinger equation (NLSE) which governs the light evolution along the optical fiber. One of the variants of the NFT-based transmission methods is the so-called eigenvalue communication in which the data is mapped on some invariant quantities (eigenvalues) of NLSE [3, 4], emerging form the NFT signal decomposition.

We consider the normalized NLSE written for the slowly varying envelop of the electromagnetic field $q(t,z)$, taking into account of noise arising due to the amplification (we assume the distributed ideal Raman amplification):

$$i\,q_z + q_{tt} + 2q|q|^2 = n, \tag{1}$$

where $n(t,z)$ is the circularly symmetric complex Gaussian noise process with $\langle n(t,z), n(t',z')\rangle = D\,\delta(t-t', z-z')$, where D depends on the signal and fiber characteristics, and $\delta(\cdot)$ is the Dirac delta-function [1, 2].

2. Nonlinear Fourier transform

For the NLSE (1), the NFT (or PNFT) signal decomposition amounts to the solution of the so-called Zakharov-Shabat system (ZSS) of differential equations [1–5]:

$$\begin{bmatrix} i\partial_t & q(t,z) \\ -q^*(t,z) & -i\partial_t \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \lambda \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}, \tag{2}$$

from which the eigenvalues, λ, and eigenfunctions, $\Phi = [\phi_1, \phi_2]^T$, are obtained. For the periodic signals with period T, $q(t+T,z) = q(t,z)$, the part of the PNFT spectrum is given by the ZSS eigenvalues corresponding to (anti-) periodic eigenfunctions [5]. The set of these points is called the *main spectrum*: these eigenvalues do not depend on z. So far, for most of the NFT-based transmission studies, the solutions of NLSE have been assumed to decay as $t \to \pm\infty$ [1–4]. However, due to some advantages of periodic signals, such as smaller processing windows and a better control over a time domain, we propose to use the periodic continuation of the signals and deal with the periodic NLSE [5], considering the transmission system based on the PNFT processing; see [6] for more details on the PNFT system design. Within the forward PNFT operation, we set the conditions for the ZSS (2) solution at some arbitrary base point t_0 as: $\Phi(t_0;\lambda) = [1,0]^T$ and $\tilde{\Phi}(t_0;\lambda) = [0,1]^T$. Then one defines the monodromy matrix for the system (2) as $M = [\Phi(t_0+T, t_0;\lambda), \tilde{\Phi}(t_0+T, t_0;\lambda)]^T$, from which the main spectrum is obtained via $\mathbb{M} = \{\lambda\,|\,\mathrm{Tr}M(t_0;\lambda) = \pm 2\}$ [5, 7, 8].

The invariance of the PNFT main spectrum makes it sensible to modulate the main spectrum points in a way similar to the original eigenvalue communication idea [3]: according to the data to be transmitted, a complex number is drawn from an appropriate constellation, and an NLSE solution having the corresponding point in the main spectrum is constructed (see [6] for the similar example communication system). The resulting signal is then launched into a fiber. Dispersion and nonlinearity are included into the PNFT and do not produce any detrimental impact. But the noise results in the random walks of the main spectrum points positions and, thus, can bring about the data errors due to the corruptions of the main spectrum of received signal compared to the initial one. In this work, making use of results given in [7] for constructing a solution of NLSE (in our case, the latter corresponds to a particular class of main spectra that can be used for our making up a two-dimensional data-carrying constellation of PNFT eigenvalues), we study numerically the performance of the PNFT-based system through computing its BER and spectral efficiency.

3. Signal construction and modulation using the perturbed plane wave

The main spectrum of a plane wave, $q(t,z) = q_0 e^{i\mu t}$, where q_0 is amplitude and μ is frequency, comprises one nondegenerate point and some degenerate ones [7,8]. Under small perturbations, the degenerate points can be split into two nondegenerate ones. The new signal resulting from perturbing the plane wave can now be described by new points in the main spectrum or, equivalently, by the value of separations between the new nondegenerate points. Following the procedure of Ref. [7], one can construct a signal by mapping the data onto the values of separations between the new points of the main spectrum. Our modulated signal can be written as:

$$q(t,z) = q(0,0)\,\frac{\Theta(\mathbf{W}^-|\tau)}{\Theta(\mathbf{W}^+|\tau)}\,e^{ik_0 z - i\omega_0 t}, \quad \text{where} \quad \Theta(\mathbf{W}|\tau) = \sum_{\mathbf{m}\in\mathbb{Z}^{g-1}} \exp\left(2\pi i \mathbf{m}^T \mathbf{W} + \pi i \mathbf{m}^T \tau \mathbf{m}\right). \tag{3}$$

In Eq. (3) k_0 and ω_0 are some constants obtained from the PNFT spectrum, and the Riemann theta function, $\Theta(\mathbf{W}|\tau)$, is the multidimensional generalization of the ordinary FT [5], where g is a finite number defining the number of modes in our solution, $\mathbf{m}$ is a point of the $(g-1)$-dimensional integer lattice, and $\mathbf{W}^{\pm} = \pi(\mathbf{k}z + \boldsymbol{\omega}t + \boldsymbol{\delta}^{\pm})/2$ is also calculated from the PNFT spectrum. The set $\{\mathbf{k}, \boldsymbol{\omega}, \boldsymbol{\delta}, \tau\}$ is dependent on the vector of separations (we designate the latter as $\boldsymbol{\varepsilon}$) [5]. For the perturbed plane wave, the Riemann spectrum up to the first order of $|\boldsymbol{\varepsilon}|$ is given by [5,7]:

$$\boldsymbol{k} \approx -2\sqrt{A^2 + \boldsymbol{\lambda}'}, \quad \boldsymbol{\omega} = -2\boldsymbol{\lambda}' \odot \boldsymbol{k}, \quad \boldsymbol{\delta}^{\pm} = \pi + i \ln\left[\boldsymbol{\sigma} \odot (\boldsymbol{\lambda}' \pm \frac{1}{2}\boldsymbol{k}) \odot (\boldsymbol{\lambda}' + \frac{1}{2}\mathbf{k})\right], \tag{4}$$

where the vector of Riemann sheet indices, $\boldsymbol{\sigma}$, consists of ± 1, and the sign $\odot$ means the element-wise multiplication.

4. Communication system and simulation results

In our system design, we form a QAM constellation with small enough values of separations ε_i, where $|\varepsilon_i|^2 \ll 1$ to ensure the sufficient accuracy of (4), and construct the particular signal using Eq. (3). This signal is sent into a link taking into account the ASE noise; at the receiver we filter out the out-of-band noise. Then, performing the direct PNFT at the receiver we retrieve the transmitted data from the values of separations ε_i between the signal's main spectrum points. In simulations our signal bandwidth was 6 GHz, the time duration (period) was set to be 498 ps, the signal power was -0.5 dBm, the fiber parameters used were equal to those in Ref. [2], and the ASE noise power was set as to compensate for the attenuation with the fiber loss coefficient value 0.2 dB/Km. We transmit a stream of 256 consecutive signals separated by guard interval filled by cyclic prefix (periodic signal extension) to overcome inter-symbol interference up to 1000 Km. At the receiver, the main spectrum is calculated by performing the direct transform; the typical received constellations are shown in Fig. 1. The left panel in Fig. 2 shows the BER calculated by directly counting the mismatches between transmitted and received bit stream vs. distance for different sizes of constellation. One can see that the BER increases with distance but still remains less than the FEC threshold of 0.02.
To find the mutual information one has to know the conditional probability distribution of the received eigenvalues. We calculated this PDF, $p(\hat{s}|s_i)$ numerically, where $\hat{s}$ is the calculated eigenvalue at the receiver and s_i is the transmitted one, by simulating the transmission of a single symbol in a 2000 km link with ASE noise using 10^4 runs. It turns out that to an acceptable order of approximation, for each transmitted eigenvalue, s_i, $p(\hat{s}|s_i)$ is a Gaussian PDF with the mean s_i and a variance dependent on s_i (see the right panel of Fig. 2 for the conditional PDFs of the complex PNFT eigenvalue). The first approximation would be to consider the largest variance and find an upper bound for the mutual information. However, since the two points in the main spectrum are mirrored with respect to a point on the imaginary part, it is possible to diminish the dependency of the PDFs on the transmitted eigenvalues s_i by using couples of mirrored points (as the clouds are similar among different points in Fig. 1). In this way, we are able to turn the nonlinear fiber channel into a band-limited Gaussian one which gives the spectral efficiency of 1.51 bits/s/Hz for our PNFT system.

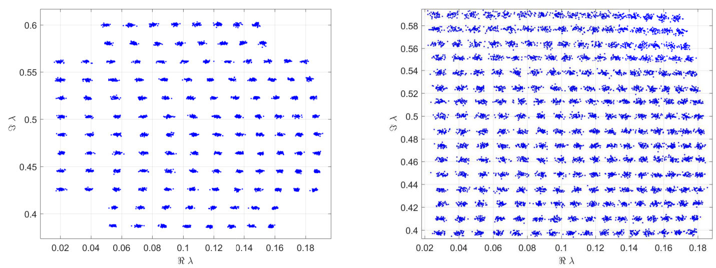

Fig. 1: Received PNFT eigenvalue constellation of sizes $K = 128, 256$.

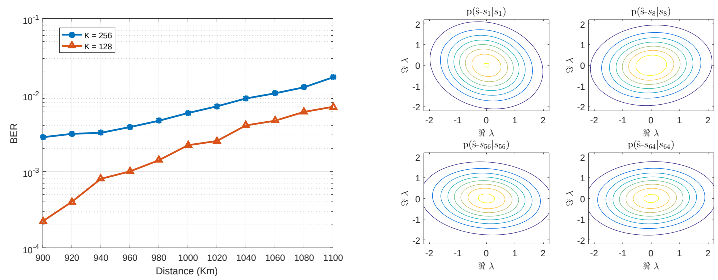

Fig. 2: Left: BER vs distance for PNFT eigenvalue constellation of size $K = 128, 256$. Right: the conditional PDFs of the received eigenvalues on transmitted ones, $p(\hat{s}|s_i)$, for different points of the constellation. With an acceptable level of approximation these PDFs are Gaussians with signal-independent variances.

5. Conclusion

We propose a fiber-optic communication system based on PNFT and assess its performance metrics by simulating the transmission in a fiber link with the account of the ASE noise. The values of the BER obtained proved a good system performance even for a relatively large number of constellation points used. Further, we calculated the spectral efficiency by calculating the empirical conditional PDF for PNFT eigenvalues. As we found that their PDFs are very close to Gaussian distribution with signal-independent variances, using the Shannon formula for band-limited signals we calculated the value of the spectral efficiency to be 1.51 bit/s/Hz.

This work was supported by the UK EPSRC Programme Grant UNLOC EP/J017582/1.

References

1. M. I. Yousefi and F. R. Kschischang, “Information Transmission Using the Nonlinear Fourier Transform, Parts I – III,” IEEE Trans. Inf. Theory, **60**, 4312–4369 (2014).
2. J. E. Prilepsky, S. A. Derevyanko, and S. K. Turitsyn, “Nonlinear spectral management: linearization of the lossless fiber channel,” Opt. Express **21**, 24344–24367 (2013).
3. A. Hasegawa and T. Nyu, “Eigenvalue communication,” J. Lightwave Technol. **11**, 395–399 (1993).
4. A. Maruta, “Eigenvalue Modulated Optical Transmission System (invited),” in Proc. of the 20th OptoElectronics and Communications Conference (OECC), Shanghai, China, Paper JThA.21 (2015).
5. A. Osborne, *Nonlinear Ocean Waves and the Inverse Scattering Transform*, 1st ed. (Academic Press, 2010).
6. M. Kamalian, J. E. Prilepsky, S. T. Le, and S. K. Turitsyn, ”Optical communication based on the periodic nonlinear Fourier transform signal processing,” IEEE 6th International Conference On Photonics (ICP), Sarawak, Malaysia (2016).
7. E. Tracy and H. Chen, “Nonlinear self-modulation: An exactly solvable model,” Phys. Rev. A **37**, 815–839 (1988).
8. S. Wahls and H. Poor, ”Fast numerical nonlinear Fourier transforms,” IEEE Trans. Inf. Theory, **61**, 6957–6974 (2015).